CONTROL SYSTEM
&
COMPONENT

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Lecturer (Electronics)

SYLLABUS

Th2. Control System and Component

	Total Periods	60
	Periods/Week	4
Examination	**Time**	**Marks**
Internal Assessment (IA)	1 hr	20
End Semester (ES)	3 hrs	80
	Total	100

Sl. No.	Topics	Periods
1	Fundamental of control system	05
2	Transfer functions	08
3	Control system components & mathematical modelling of physical system	05
4	Block diagram & signal flow graphs (SFG)	08
5	Time domain analysis of control systems	08
6	Feedback characteristics of control systems	06
7	Stability concept, & root locus method	08
8	Frequency-response analysis & Bode plot	07
9	State variable analysis	05
	Total	**60**

Course Contents:

1. **Fundamentals of control System**

   1.1. Classification of Control system
   1.2. Open loop system & Closed loop system and its comparison
   1.3. Effects of Feed back
   1.4. Standard test Signals (Step, Ramp, Parabolic, Impulse Functions)
   1.5. Servomechanism
   1.6. Regulators (Regulating systems)

2. **Transfer Function**

   2.1. Transfer Function of a system & Impulse response,
   2.2. Properties, Advantages & Disadvantages of Transfer Function
   2.3. Poles & Zeroes of transfer Function
   2.4. Representation of poles & Zero on the s-plane

2.5. Simple problems of transfer function of network

3. **Control system Components & mathematical modelling of physical System**

   3.1. Components of Control System
   3.2. Potentiometer, Synchro, Diode modulator & demodulator
   3.3. DC motors, AC Servomotors
   3.4. Modelling of Electrical Systems (R, L, C, Analogous systems)

4. **Block Diagram & Signal Flow Graphs (SFG)**

   4.1. Definition of Basic Elements of a Block Diagram
   4.2. Canonical Form of Closed loop Systems
   4.3. Rules for Block diagram Reduction
   4.4. Procedure for of Reduction of Block Diagram
   4.5. Simple Problem for equivalent transfer function
   4.6. Basic Definition in SFG & properties
   4.7. Mason's Gain formula
   4.8. Steps foe solving Signal flow Graph
   4.9. Simple problems in Signal flow graph for network

5. **Time Domain Analysis of Control Systems**

   5.1. Definition of Time, Stability, steady-state response, accuracy, transient accuracy, In-sensitivity and robustness.
   5.2. System Time Response
   5.3. Analysis of Steady State Error
   5.4. Types of Input & Steady state Error (Step, Ramp, Parabolic)
   5.5. Parameters of first order system & second-order systems
   5.6. Derivation of time response Specification (Delay time, Rise time, Peak time, Setting time, Peak overshoot)

6. **Feedback Characteristics of Control Systems**

   6.1. Effect of parameter variation in Open loop System & Closed loop Systems
   6.2. Introduction to Basic control Action& Basic modes of feedback control: proportional, integral and derivative
   6.3. Effect of feedback on overall gain, Stability
   6.4. Realisation of Controllers (P, PI, PD, PID) with OPAMP

7. **Stability concept& Root locus Method**

   7.1. Effect of location of poles on stability
   7.2. Routh Hurwitz stability criterion.
   7.3. Steps for Root locus method
   7.4. Root locus method of design (Simple problem)

8. **Frequency-response analysis** & **Bode Plot**

   8.1. Frequency response, Relationship between time & frequency response
   8.2. Methods of Frequency response

8.3. Polar plots & steps for polar plot
8.4. Bodes plot & steps for Bode plots
8.5. Stability in frequency domain, Gain Margin& Phase margin
8.6. Nyquist plots. Nyquist stability criterion.
8.7. Simple problems as above

9. **State variable Analysis**

9.1. Concepts of state, state variable, state model
9.2. State models for linear continuous time functions (Simple)

LESSON PLAN

CONTROL SYSTEM AND COMPONENT (TH2) - 6TH SEMESTER ETC		
Week	**No of Periods Allotted (60)**	**Syllabus To be Covered**
1ST	**1.Fundamental of Control System - 5P**	
	1st	1.1 Classification of Control system
	2nd	1.2 Open loop system & Closed loop system and its comparison
	3rd	1.3 Effects of Feed back
	4th	1.4 Standard test Signals (Step, Ramp, Parabolic, Impulse Functions)
2ND	**1st**	1.5 Servomechanism
	2. Transfer Functions - 8P	
	2nd	2.1 Transfer Function of a system & Impulse response,
	3rd	2.2 Properties, Advantages & Disadvantages of Transfer Function
	4th	2.3 Poles & Zeroes of transfer Function
3RD	**1st**	2.4 Poles & Zeroes of transfer Function
	2nd	2.5 Representation of poles & Zero on the s-plane
	3rd	2.6 Simple problems of transfer function of network
	4th	2.6 Simple problems of transfer function of network
4TH	**1st**	2.6 Simple problems of transfer function of network
	3. Control system Components & mathematical modelling of physical System - 5P	
	2nd	3.1 Components of Control System
	3rd	3.2 Potentiometer, Synchro, Diode modulator & demodulator
	4th	3.2 Potentiometer, Synchro, Diode modulator & demodulator
5TH	**1st**	3.3 DC motors, AC Servomotors
	2nd	3.4 Modelling of Electrical Systems (R, L, C, Analogous systems)
	4. Block Diagram & Signal Flow Graphs (SFG) - 8P	
	3rd	4.1 Definition of Basic Elements of a Block Diagram
	4th	4.2 Canonical Form of Closed loop Systems
6TH	**1st**	4.3 Rules for Block diagram Reduction 4.4 Procedure for of Reduction of Block Diagram
	2nd	4.5 Simple Problem for equivalent transfer function
	3rd	4.6 Basic Definition in SFG & properties
	4th	4.7 Mason's Gain formula
7TH	**1st**	4.8 Steps foe solving Signal flow Graph
	2nd	4.9 Simple problems in Signal flow graph for network
	5. Time Domain Analysis of Control Systems - 8P	
	3rd	5.1 Definition of Time, Stability, steady-state response, accuracy, transient accuracy, In-sensitivity and robustness.
	4th	5.2 System Time Response

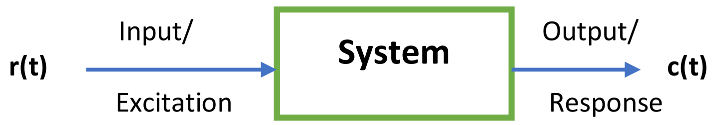
8TH	**1st**	5.3 Analysis of Steady State Error
	2nd	5.4 Types of Input & Steady state Error(Step ,Ramp, Parabolic)
	3rd	5.5 Parameters of first order system & second-order systems
	4th	5.6 Derivation of time response Specification (Delay time, Rise time, Peak time, Setting time, Peak over shoot)
9TH	**1st**	5.6 Derivation of time response Specification (Delay time, Rise time, Peak time, Setting time, Peak over shoot)
	2nd	5.6 Derivation of time response Specification (Delay time, Rise time, Peak time, Setting time, Peak over shoot)
	6. Feedback Characteristics of Control Systems - 6P	
	3rd	6.1 Effect of parameter variation in Open loop System & Closed loop Systems
	4th	6.2 Introduction to Basic control Action& Basic modes of feedback control: proportional, integral and derivative
10TH	**1st**	6.3 Effect of feedback on overall gain, Stability
	2nd	6.3Effect of feedback on overall gain, Stability
	3rd	6.4 Realisation of Controllers (P, PI, PD, PID) with OPAMP
	4th	6.4 Realisation of Controllers (P, PI, PD, PID) with OPAMP
11TH	**7. Stability concept& Root locus Method - 8P**	
	1st	7.1 Effect of location of poles on stability
	2nd	7.2 Routh Hurwitz stability criterion.
	3rd	7.3 Routh Hurwitz stability criterion.
	4th	7.3 Routh Hurwitz stability criterion.
12TH	**1st**	7.4 Steps for Root locus method
	2nd	7.5 Root locus method of design (Simple problem)
	3rd	7.5 Root locus method of design (Simple problem)
	4th	7.5 Root locus method of design (Simple problem)
13TH	**8. Frequency-response analysis & Bode Plot -7P**	
	1st	8.1 Frequency response, Relationship between time & frequency response
	2nd	8.2 Methods of Frequency response
	3rd	8.3 Polar plots & steps for polar plot
	4th	8.4 Bodes plot & steps for Bode plots
14TH	**1st**	8.5 Stability in frequency domain, Gain Margin& Phase margin
	2nd	8.6 Nyquist plots. Nyquist stability criterion.
	3rd	8.7 Simple problems as above
	9. State variable Analysis - 5P	
	4th	9.1 Concepts of state, state variable, state model,
15TH	**1st**	9.1 Concepts of state, state variable, state model,
	2nd	9.2 state models for linear continuous time functions (Simple)
	3rd	9.2 state models for linear continuous time functions (Simple)
	4th	9.2 state models for linear continuous time functions (Simple)

1. Fundamentals of Control System

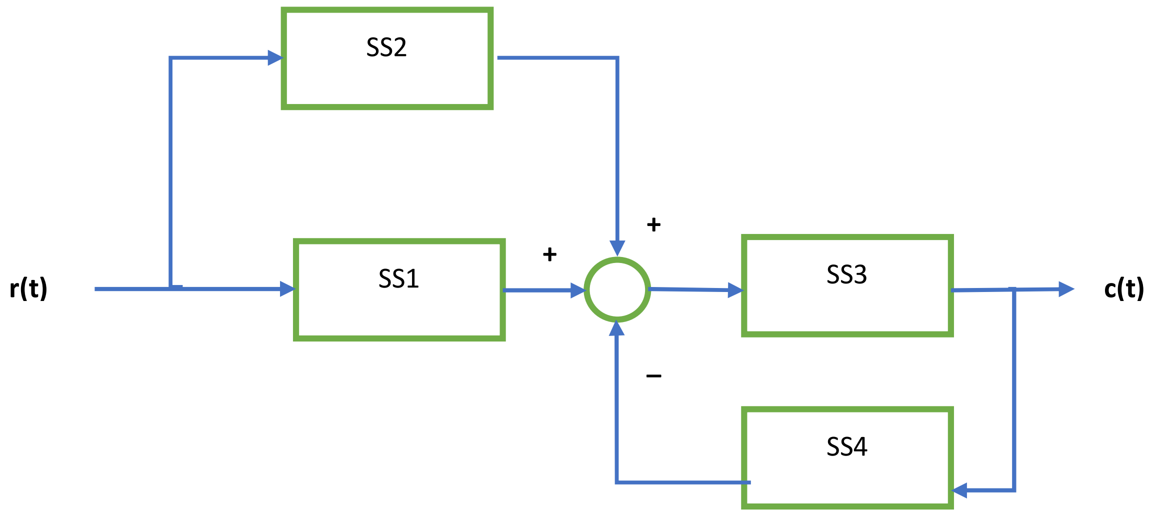
1.1. Classification of Control System

System:

- A system is a combination of components (physical, biological or abstract) which together perform an intended objective.
- A system gives an output (response) for an input (excitation).



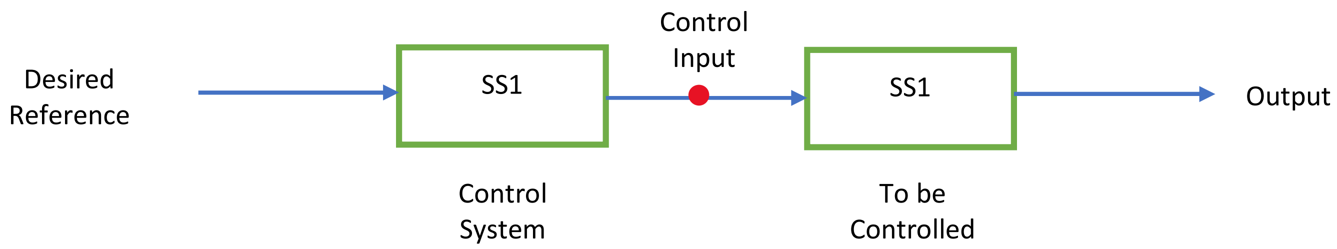
- A system can be a collection of multiple subsystems.



- Example of Systems
  - **Motor**:
    Input: Electrical energy (Voltage)
    Output: Mechanical Energy (Torque)
  - **Vehicle**:
    Input: Acceleration/Deceleration
    Output: Displacement

Control System:

- A system which directs the input to other systems or regulates its output is called a control system.
- Control system alters the response of a system as desired.



Classification of systems:

Some of the important classifications of systems are

a. Linear and Non-Linear Systems
b. Static and Dynamic Systems
c. Time variant and Time invariant systems
d. Causal and non-causal systems

a. Linear System	Non – linear system
Output of the system varies linearly with input	Output of the system does not vary linearly with time.
Satisfies superposition and superposition principle.	Does not satisfy superposition and superposition principle.
Example: Resistor $$\boldsymbol{R = \frac{V}{I}}$$	Example: Diode $$I = I_0 e^{(\frac{qV}{kT}-1)}$$

b. Static System	Dynamic System
At any time, output of the system depends only on present input.	Output of the system depends on present as well as past inputs.
Memory less system	Presence of memory can be observed
$\boldsymbol{y(t) = f(u(t))}$	$y(t) = f(u(t), u(t-1), u(t-2), \ldots\ldots)$
Example: Resistor $$\boldsymbol{I(t) = \frac{V(t)}{R(t)}}$$	Example: Inductor $$I(t) = \frac{1}{L}\int_0^t V(t)dt$$

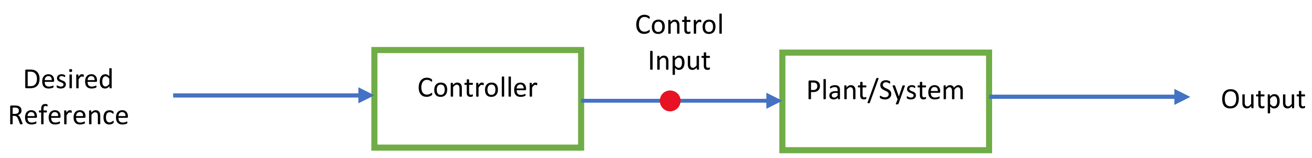
C. Time variant	Time invariant
Output of the system is independent of the time at which the input is applied.	Output of the system depends on the time at which the input is applied.
$\boldsymbol{y(t) = f(u(t)) \Longrightarrow y(t+\delta) = f(u(t+\delta))}$	$y(t) = f(u(t)) \not\Rightarrow y(t+\delta) = f(u(t+\delta))$
Example: An ideal Resistor $\boldsymbol{I(t) = \frac{V(t)}{R} \Longrightarrow I(t+\delta) = \frac{V(t+\delta)}{R}}$	Example: Aircraft Mass of aircraft changes as fuel is consumed Acceleration, $a(t) = \frac{F(t)}{M(t)}$

D. Causal System	Non Causal System
Output depends only on the inputs already received (present or past).	Output depends on future inputs as well.
Non anticipatory system.	System anticipates future inputs based on past.
$\boldsymbol{y(t) = f(x(t), x(t-1), \ldots)}$	$y(t) = f(x(t), x(t+1), \ldots)$
Example: Motor or Generator	Example: Weather forecasting system

1.2. Open Loop and Closed Loop System

<u>Open Loop Control System:</u>

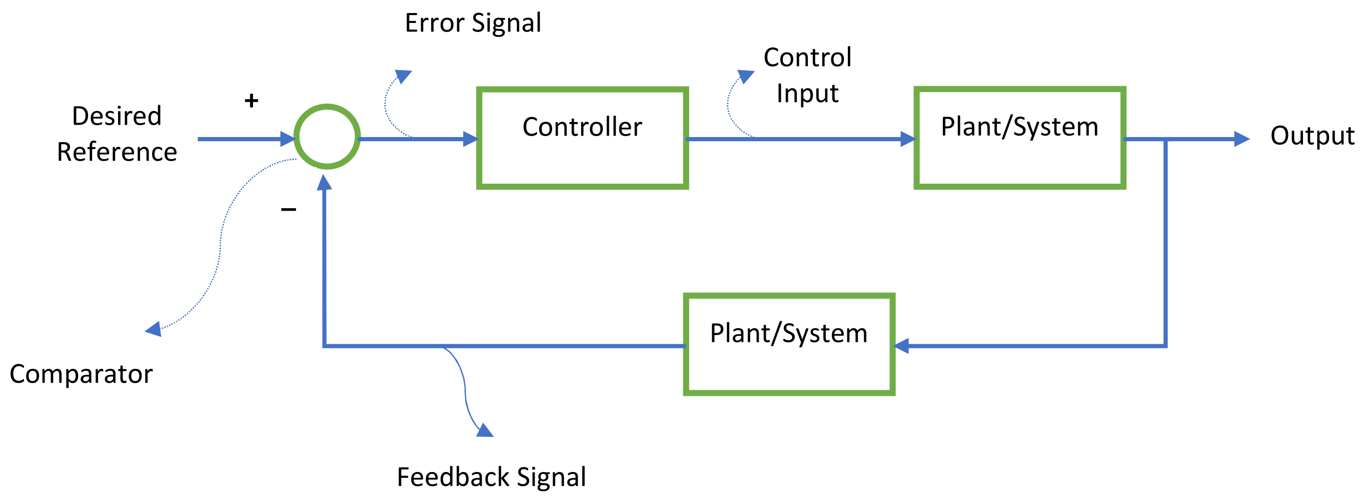
- It is a control system in which output has no effect on the controller action.
- Example: Traffic Light, Washing Machine, Bread toaster etc.



- Advantages:
    I. Simple design and easy to construct.
    II. Economical.
    III. Easy maintenance.
    IV. Highly Stable.
- Disadvantages:
    I. Not accurate and not reliable when system parameters vary.
    II. Recalibration is needed in regular interval.

<u>Closed Loop Control System:</u>

- It is a control system in which controller action is affected by output.
- Example: Automatic electric iron, Speed control of dc motor, Missile launching system



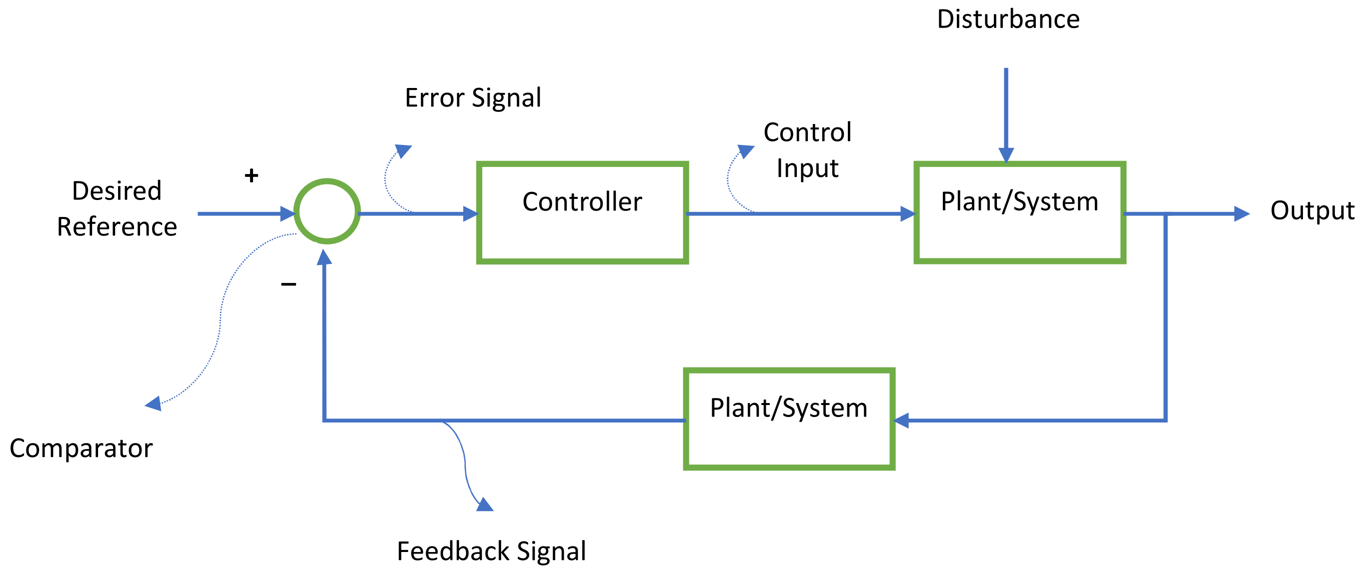
- **<u>Advantages:</u>**
    I. Can operate efficiently even when system parameters vary.
    II. Less non-linearity effect of these systems on output.
    III. High bandwidth of operation
    IV. Provision of automation
    V. Time to time calibration of parameters is not required.
- **<u>Disadvantages:</u>**
    I. Complex design
    II. More expensive
    III. Difficulty in maintenance
    IV. Less stable than open loop control system

<u>Comparison of open loop and closed loop control system:</u>

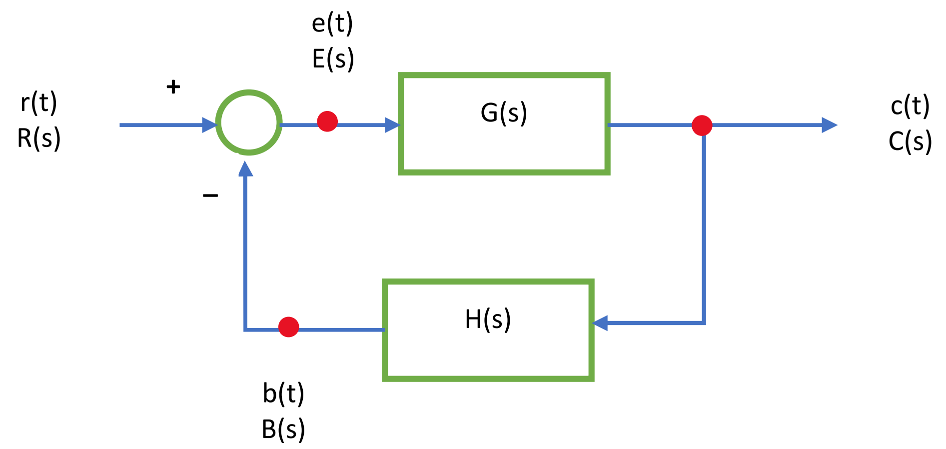
Basis for comparison	Open Loop Control System	Closed Loop Control System
Definition	It is a control system in which output has no effect on the controller action.	It is a control system in which controller action is affected by output.
Other name	Non feedback system	Feed back system
Components	Controller, Controlled Process	Amplifier, Controller, Controlled Process, Feedback
Construction	Simple	Complex
Reliability	Non reliable	Reliable
Accuracy	Depends on calibration	Accurate because of feedback
Stability	Stable	Less stable
Optimization	Not possible	Possible
Response	Fast	Slow
Calibration	Difficult	Easy
System disturbance	Affected	Not affected
Linearity	Non linear	Linear
Example	Traffic light, Automatic Washing machine, Immersion Rod, TV Remote	Air Conditioner, Refrigerator, Toaster.

1.3. Effect of Feedback

- Feedback system senses the plant (system) output and gives a feedback signal which can be compared with desired refence.
- Controller action changes based on the feedback signal.
- Feedback enables the control system in extracting the desired performance from the plant even in the presence of disturbance.



Standard Negative Feedback System:



$C(s) = E(s)G(s)$ ---------------*(1)*

$B(s) = C(s)H(s)$ ---------------*(2)*

$E(s) = R(s) - B(s)$ ---------------*(3)*

Put (2) & (3) in (1)

$$C(s) = [R(s) - C(s)H(s)]G(s)$$

$$\Longrightarrow C(s) = R(s)G(s) - C(s)H(s)G(s)$$

$$\Longrightarrow C(s)[1 + G(s)H(s)] = G(s)R(s)$$

$$\Longrightarrow \boxed{\frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}} = \frac{\boldsymbol{G(s)}}{\boldsymbol{1 + G(s)H(s)}}}$$

Closed Loop Transfer Function (CLTF) of standard negative feedback control system is

$$\boxed{\boldsymbol{T(s)} = \frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}} = \frac{\boldsymbol{G(s)}}{\boldsymbol{1 + G(s)H(s)}}}$$

And the CLTF of a standard positive feedback control system is

$$\boxed{\boldsymbol{T(s)} = \frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}} = \frac{\boldsymbol{G(s)}}{\boldsymbol{1 - G(s)H(s)}}}$$

<u>Sensitivity:</u>

$$ {}_{G}^{T}S = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{G}{T}\frac{\partial T}{\partial G}$$

I.<u>For -ve feedback system:</u>

$$ {}_{G}^{T}S = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{G}{T}\frac{\partial T}{\partial G} = \frac{G}{\frac{G}{1+GH}}\frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right)$$

$$= (1 + GH)\left[\frac{1.(1+GH)-GH}{(1+GH)^2}\right]$$

$$= \frac{1}{1+GH}$$

$$\boxed{{}_{G}^{T}\boldsymbol{S} = \frac{\boldsymbol{1}}{\boldsymbol{1 + GH}}}$$

II.<u>For +ve feedback system:</u>

Similarly for standard positive feedback system,

$$\boxed{{}^{T}_{G}S = \frac{1}{1-GH}}$$

<u>Effect of feedback on control system:</u>

Due to feedback following factors of a control system are affected

- Overall gain
- Stability
- Sensitivity

<u>Overall Gain:</u>

-ve feedback	$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$	Gain decreases
+ve feedback	$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$	Gain increases

<u>Stability:</u>

$Stability \propto Bandwidth$

$Gain \times Bandwidth = Constant$

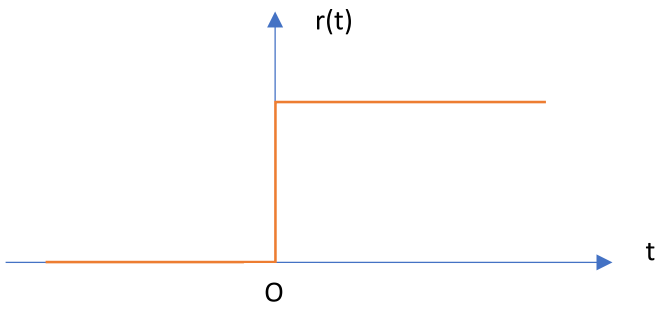
-ve feedback	Gain decreases	Bandwidth Increases	Stability increases
+ve feedback	**Gain increase**	**Bandwidth decreases**	**Stability decreases**

<u>Sensitivity:</u>

-ve feedback	$\boxed{{}^{T}_{G}S = \frac{1}{1+GH}}$	Sensitivity decreases
+ve feedback	${}^{T}_{G}S = \frac{1}{1-GH}$	Sensitivity increases

1.4. <u>Standard test Signals (Step, Ramp, Parabolic, Impulse Functions)</u>

- **<u>Step Signal:</u>**



$r(t) = Au(t)$

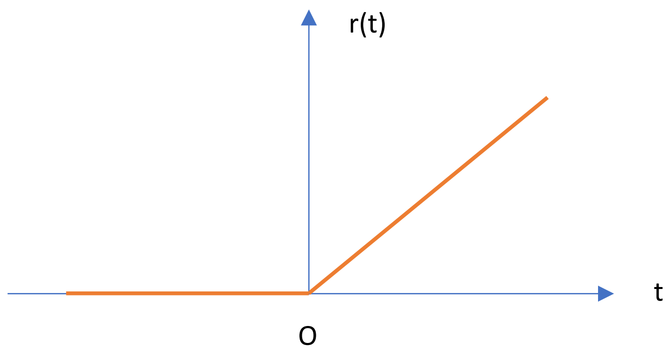
where,

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

u(t) is the unit step signal.
At t=0 there is no analysis as initial conditions are ignored.

$$\boxed{R(s) = \frac{A}{s}}$$

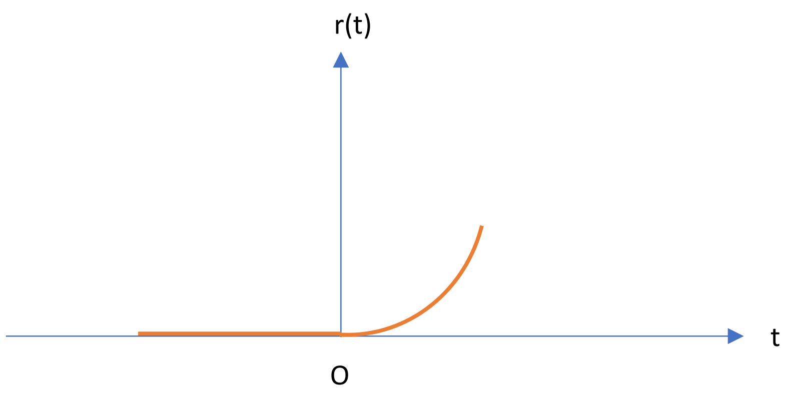
- **<u>Ramp Signal</u>**



$r(t) = Atu(t)$

$$\boxed{R(s) = \frac{A}{s^2}}$$

- <u>Parabolic Signal:</u>



$r(t) = A\frac{t^2}{2}u(t)$

$$\boxed{R(s) = \frac{A}{s^3}}$$

1.5. <u>Servomechanism</u>

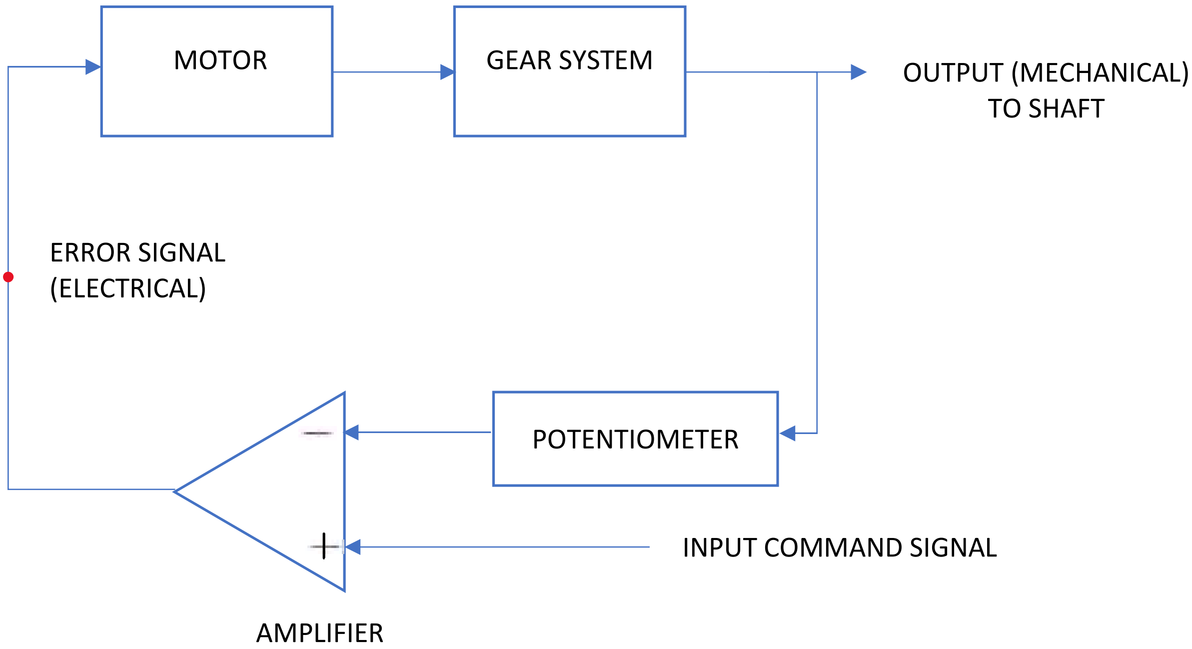
Link for a simple servomechanism example:

https://youtu.be/PMFDb3k9Gsw

A servomechanism may be defined as a power amplifying device in which the amplifying element driving the output is actuated by the difference between the input to the servo and its output.

Components of servomechanism

I. A controlled device
II. An output sensor
III. A feedback system



- Servomechanism is an automatic closed loop control system.
- The device is controlled by feedback signal generated by comparing output signal and reference input signal.
- Here, the input command signal is electrical and the output is mechanical in nature. The output sensor (transducer in this case) converts the output into its equivalent electrical signal.
- When the feedback signal (or error signal) signal becomes zero, the controlled system will produce no output to drive the shaft

4. FREQUENCY RESPONSE ANALYSIS

4.1 Correlation between time response and frequency response

Frequency-Domain Specifications :-

The following frequency-domain specifications are often used :

(a) Resonant Peak (M_r) :

The resonant peak M_r is the maximum value of $|M(j\omega)|$.

(b) Resonant Frequency (ω_r) :

The resonant frequency ω_r is the frequency at which the peak resonance M_r occurs.

(c) Bandwidth (BW) :

The bandwidth (BW) is the frequency at which $|M(j\omega)|$ drops to 70.7% of, or 3 dB down from, its zero frequency value.

M_r, ω_r and BW of the prototype second order system

Consider the closed-loop transfer function (CLTF) of the prototype second order system.

$$M(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{--- (1)}$$

At sinusoidal steady state $s = j\omega$, eqn. (1) becomes

$$M(j\omega) = \frac{Y(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2} \quad \text{--- (2)}$$

$$= \frac{\omega_n^2}{-\omega^2 + j2\xi\omega\omega_n + \omega_n^2}$$

$$= \frac{1}{-\left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\left(\frac{\omega}{\omega_n}\right) + 1}$$

Let $u = \frac{\omega}{\omega_n}$,

(2) becomes:

$$M(ju) = \frac{1}{1 + j2\xi u - u^2} \quad \text{— (3)}$$

The magnitude and phase of (3) are:

$$|M(ju)| = \frac{1}{\left[(1-u^2)^2 + (2\xi u)^2\right]^{1/2}}$$

and $\angle M(ju) = \phi_M(ju) = -\tan^{-1}\left[\frac{2\xi u}{1-u^2}\right]$

The resonant frequency of system is determined by setting $\frac{d|M(ju)|}{du} = 0$

$$\Rightarrow \frac{d}{du}\left[\frac{1}{\left[(1-u^2)^2 + (2\xi u)^2\right]^{1/2}}\right] = 0$$

$$\Rightarrow -\frac{1}{2}\left[(1-u^2)^2 + (2\xi u)^2\right]^{-3/2}\left[2(1-u^2)(0-2u) + 2(2\xi u)(2\xi)\right] = 0$$

$$\Rightarrow \left[4u^3 - 4u + 8\xi^2 u\right] = 0$$

$$\Rightarrow 4u\left[u^2 - 1 + 2\xi^2\right] = 0$$

$$\therefore \quad u = 0 \quad \text{or} \quad u = \sqrt{1 - 2\xi^2}$$

Normalized resonant frequency is: $\frac{\omega_r}{\omega_n} = u_r = \sqrt{1 - 2\xi^2}$

and the resonant frequency is: $\omega_r = \omega_n\sqrt{1 - 2\xi^2}$.

Since frequency is a real quantity.

$$2\xi^2 \le 1$$

$$\Rightarrow \xi < \frac{1}{\sqrt{2}}$$

$$\text{or} \quad \boxed{\xi \le 0.707}$$

For all values of $\xi > 0.707$, $\boxed{\omega_r = 0 \text{ and } M_r = 1}$

$\therefore$ Resonant peak $M_r = \left| M(j\omega) \right|_{\omega = \omega_r = \sqrt{1-2\xi^2}}$

$$= \frac{1}{\left[(1-\omega_r^2)^2 + (2\xi\omega_r)^2\right]^{1/2}}$$

$$= \frac{1}{\left[(1-1+2\xi^2)^2 + 4\xi^2(1-2\xi^2)\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi^4 + 4\xi^2 - 8\xi^4\right]^{1/2}}$$

$$= \frac{1}{\left[4\xi^2 - 4\xi^4\right]^{1/2}}$$

$$= \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\therefore \boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}, \quad \xi \le 0.707$$

Band Width (BW)

In accordance with the definition of bandwidth,

$$|M(ju)| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{1}{[(1-u^2)^2 + (2\xi u)^2]^{1/2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow [(1-u^2)^2 + (2\xi u)^2]^{1/2} = \sqrt{2}$$

$$\Rightarrow (1-u^2)^2 + (2\xi u)^2 = 2$$

$$\Rightarrow 1 + u^4 - 2u^2 + 4\xi^2 u^2 - 2 = 0$$

$$\Rightarrow u^4 + (4\xi^2 - 2)u^2 - 1 = 0$$

Let $u^2 = m$

$$\therefore m^2 + (4\xi^2 - 2)m - 1 = 0$$

$$\therefore m = \frac{-(4\xi^2-2) \pm \sqrt{(4\xi^2-2)^2 + 4}}{2}$$

$$= (1-2\xi^2) \pm \sqrt{4\xi^4 - 4\xi^2 + 2}$$

Considering +ve sign.

$$u = \sqrt{(1-2\xi^2) + \sqrt{(1-2\xi^2)^2 + 1}}$$

$\therefore$ Band width

$$\boxed{BW = \omega_n \sqrt{(1-2\xi^2) + \sqrt{(1-2\xi^2)^2 + 1}}}$$

Correlation between time-domain & Frequency Domain specifications :-

$$r(t) \longrightarrow \boxed{\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}} \longrightarrow y(t)$$

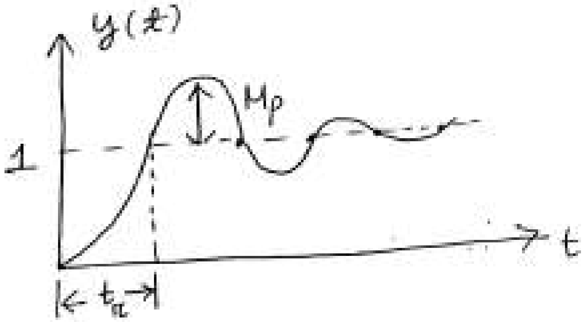
Prototype Second Order System.

Time Domain Specifications

$$M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}, \quad 0<\xi<1$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$t_r = \frac{\pi-\phi}{\omega_d} = \frac{\pi-\cos^{-1}(\xi)}{\omega_n\sqrt{1-\xi^2}}$$



$$y(t) = 1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi), \quad t\geq 0$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

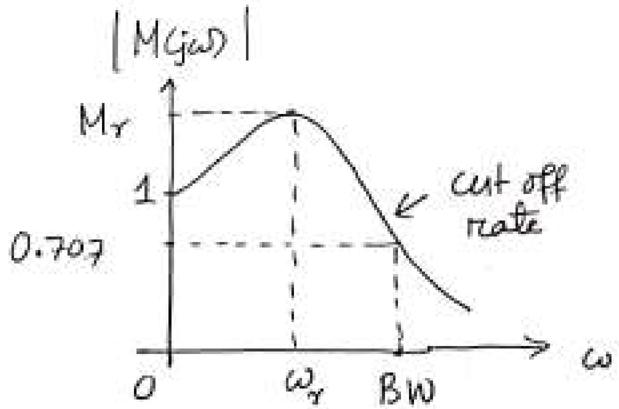
$$\cos\phi = \xi$$

Frequency Domain Specifications

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} \qquad \xi \leq 0.707$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$BW = \omega_n\sqrt{(1-2\xi^2)+\sqrt{(1-2\xi^2)^2+1}}$$



→ As ω_n gets larger, t_r gets smaller and the system responds faster.

→ As ξ gets larger, t_r gets larger and the system responds slower.

→ As ω_n gets larger, BW gets larger.

→ As ξ gets larger, BW gets smaller.

Finally,

→ Bandwidth and risetime are inversely proportional

→ Increasing ω_n, increases BW and decreases t_r

→ Increasing ξ, decreases BW and increases t_r.

---o---

Questions :

Q. ① Derive the expressions for following frequency domain specifications for a proto-type second order system:-

i) Resonant Frequency

ii) Resonant Peak.

iii) Band width

Q. ② Write down the correlation between time-domain and frequency domain specifications for a proto-type second order system.

---o---

4.2 POLAR PLOT

→ Two methods of determining stability of linear SISO system : Routh-Hurwitz Criteria, Root-Locus method. → based on locating the roots of characteristics equation in the s-plane.

→ The Nyquist Criterion is a semi-graphical method that determines the stability of a closed loop system by investigating the properties of the frequency domain plot, the Nyquist plot, of the open-loop transfer function $G(s)H(s)$.

→ Nyquist plot is plot $G(j\omega)H(j\omega)$ in the polar coordinates of $Im[G(j\omega)H(j\omega)]$ vs. $Re[G(j\omega)H(j\omega)]$ as ω varies from 0 to ∞. That is why the Nyquist plot as ω varies from 0 to ∞ is known as Polar plot.

Features of Polar Plot :-

1. Polar plot/Nyquist Plot gives information on the relative stability of the stable system, and the degree of instability of an unstable system. It gives indication on how system stability may be improved.
2. Polar plot/Nyquist Plot is very easy to obtain, especially with the aid of a computer.
3. The Polar plot of $G(s)H(s)$ gives information on the frequency domain characteristics such as M_r, ω_r, BW
4. Nyquist/Polar plot is useful for systems with pure

time delay. that cannot be treated with the Routh-Hurwitz Criterion and are difficult to analyze with Root-Locus method.

(5) Unlike the Root-locus method Nyquist criterion doesnot give the exact location of the characteristics equation roots.

Let us consider the CLTF of a SISO system.

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Where $G(s)H(s)$: OLTF can assume the following form:

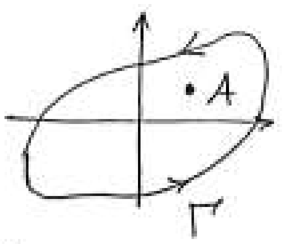
$$G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\cdots(1+T_m s)}{s^p(1+T_a s)(1+T_b s)\cdots(1+T_n s)} e^{-T_d s}$$

Where the T's are real or complex conjugate coefficients and T_d is a real time delay.

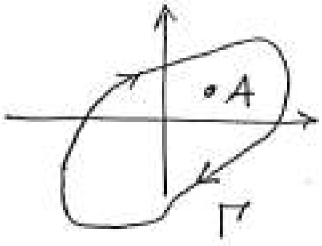
Roots of the characteristics equation are also zeros of $1+G(s)H(s)=0$

$\therefore$ Closed loop transfer Function poles $\triangleq$ Zeros of $1+G(s)H(s)=0$
$\triangleq$ Roots of characteristics Equation.

<u>Concept of Enclosement</u> :- [Convention used here]



Point A is not enclosed by contour Γ



Point A is enclosed by contour Γ

A point is said to be enclosed by a contour or closed path if it is to the right hand side of direction of contour.

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Critical Point :-

Characteristics equation,

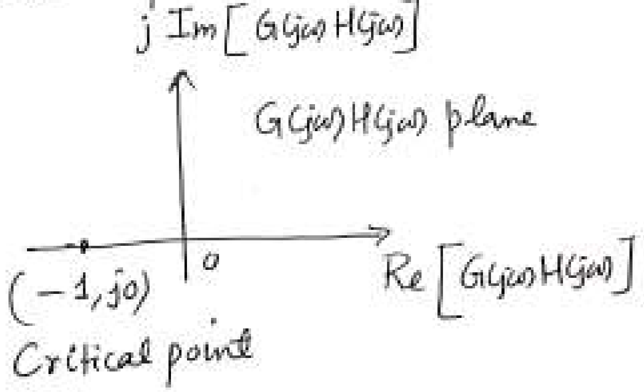
$$1 + G(s)H(s) = 0$$

$$\Rightarrow G(s)H(s) = -1$$

Put $s = j\omega$

$$G(j\omega)H(j\omega) = -1 = -1 + j0$$

The critical point is $(-1, j0)$ in the $G(j\omega)H(j\omega)$ plane.



Closed Loop Stability From Polar Plot

A closed loop system is said to be absolute stable if the polar plot doesnot enclose the critical point $(-1, j0)$

If the plot encloses the critical point, the ~~plot be~~ closed loop system becomes unstable.

Steps to be followed to determine closed loop system stability from polar plot

(i) Draw the polar plot

(ii) Determine the point of intersection of polar plot with Real axis

(iii) Locate the critical point $(-1, j0)$

(iv) Check if the critical point is enclosed by polar plot.

(v) not enclosed → Close loop stable; enclosed – close loop unstable

Q) Determine the range of values of K for which the system having OLTF

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$ is stable.

Soln:- Given OLTF :

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

$$= \frac{K}{2s(1+s)(1+\frac{1}{2}s)}$$

$$= \frac{(\frac{K}{2})}{s(1+s)(1+\frac{1}{2}s)}$$ (Time constant Form)

$$= \frac{K_1}{s(1+s)(1+\frac{1}{2}s)} \quad (\because K_1 = \frac{K}{2})$$

$$= K_1 \, G_1(s)H_1(s)$$

Let $G_1(s)H_1(s) = \dfrac{1}{s(1+s)(1+\frac{1}{2}s)}$

We draw the polar plot of $G_1(s)H_1(s)$

Put $s = j\omega$

$$G_1(j\omega)H_1(j\omega) = \frac{1}{j\omega(1+j\omega)(1+\frac{1}{2}j\omega)} \quad \text{---} (1)$$

$$|G_1(j\omega)H_1(j\omega)| = \left| \frac{1}{j\omega(1+j\omega)(1+\frac{1}{2}j\omega)} \right| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+\frac{1}{4}\omega^2)}}$$

$$\angle G_1(j\omega)H_1(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(\tfrac{1}{2}\omega)$$

ω	$\lvert G_1(j\omega)H_1(j\omega)\rvert = \dfrac{1}{\omega\sqrt{(1+\omega^2)(1+\omega^2/4)}}$	$\angle G_1(j\omega)H_1(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{2}\right)$
0	∞	-90°
∞	0	-270°

ω	$Re[G_1(j\omega)H_1(j\omega)] = \dfrac{-3/2}{(1+\omega^2)(1+\omega^2/4)}$	$Im[G_1(j\omega)H_1(j\omega)] = \dfrac{-(1-\omega^2/2)}{\omega(1+\omega^2)(1+\omega^2/4)}$
0	$-3/2$	$-\infty$
∞	-0	$+0$

From ①
$$G_1(j\omega)H_1(j\omega) = \frac{(1+j0)}{(0+j\omega)(1+j\omega)(1+\frac{1}{2}j\omega)} = \frac{(0-j\omega)(1-j\omega)(1-\frac{1}{2}j\omega)}{\omega^2(1+\omega^2)(1+\frac{\omega^2}{4})}$$

$$= \frac{(-j\cancel{\omega})(1-j\frac{3}{2}\omega-\frac{\omega^2}{2})}{\omega^{\cancel{2}}(1+\omega^2)(1+\omega^2/4)}$$

$$= \frac{-3/2\,\omega - j(1-\frac{\omega^2}{2})}{\omega(1+\omega^2)(1+\omega^2/4)}$$

$$= \frac{-\frac{3}{2}\cancel{\omega}}{\cancel{\omega}(1+\omega^2)(1+\frac{\omega^2}{4})} - j\frac{(1-\frac{\omega^2}{2})}{\omega(1+\omega^2)(1+\frac{\omega^2}{4})}$$

$$\therefore\ Re[G_1(j\omega)H_1(j\omega)] = \frac{-3/2}{(1+\omega^2)(1+\frac{\omega^2}{4})}$$

$$Im[G_1(j\omega)H_1(j\omega)] = \frac{-(1-\frac{\omega^2}{2})}{\omega(1+\omega^2)(1+\frac{\omega^2}{4})}$$

On Real axis $Im[G_1(j\omega)H_1(j\omega)] = 0$

$$\Rightarrow 1-\frac{\omega^2}{2} = 0$$

$$\Rightarrow \frac{\omega^2}{2} = 1 \Rightarrow \omega^2 = 2 \Rightarrow \boxed{\omega = \sqrt{2}}$$

And
$$Re[G_1(j\omega)H_1(j\omega)]\Big|_{\omega=\sqrt{2}} = \frac{-3/2}{(1+2)(1+\frac{2}{4})} = -\frac{1}{3}$$

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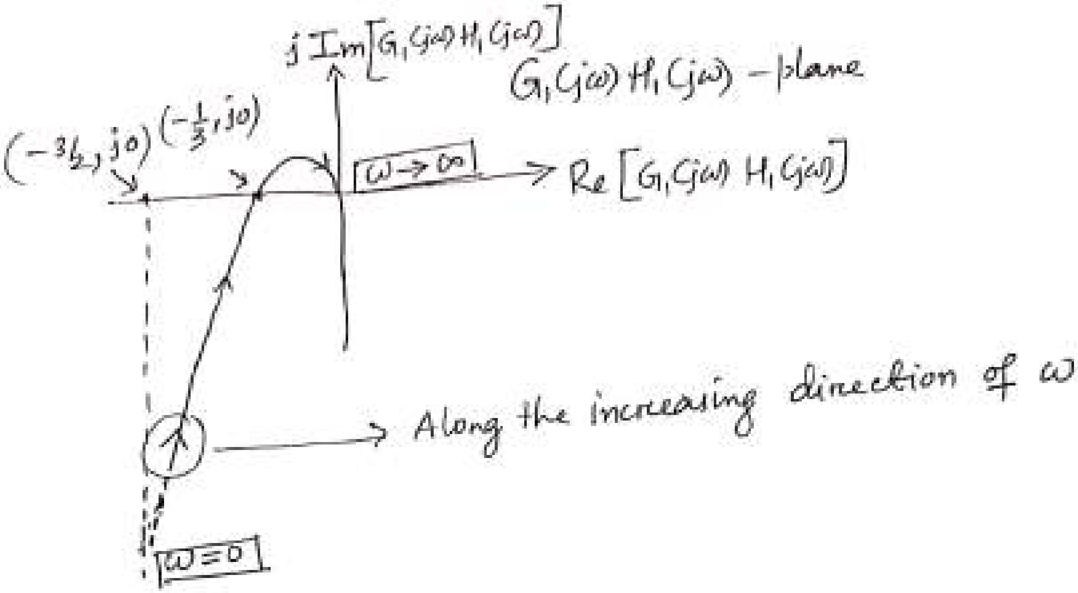
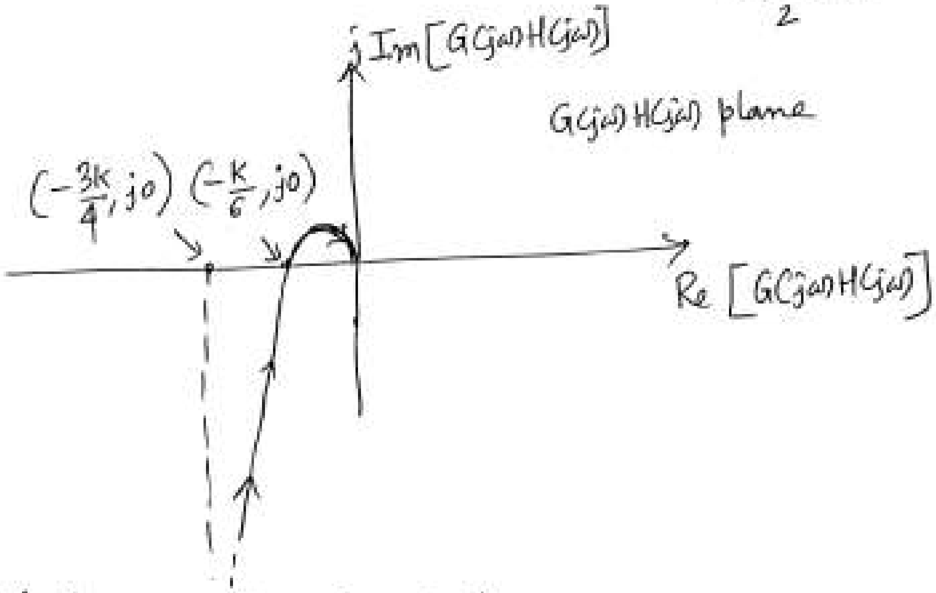


Polar plot of $G(s)H(s) = K_1 G_1(s)H_1(s)$

$$= \frac{K}{2} G_1(s)H_1(s)$$



Closed loop Stability of System

Absolute Stable

→ Critical point shouldnot be enclosed

→ $(-1, j0)$ should lie to the LHS of plot

$-\frac{K}{6} > -1$

$\Rightarrow \frac{K}{6} < 1$

$\Rightarrow \boxed{K < 6}$

Marginally Stable

→ Critical point should lie on the plot

→ $(-1, j0)$ should lie on the plot

$-\frac{K}{6} = -1$

$\Rightarrow K = 6$

$\Rightarrow \boxed{K = 6}$

Unstable

→ Critical point should be enclosed

→ $(-1, j0)$ should lie to the RHS of plot

$-\frac{K}{6} < -1$

$\Rightarrow \frac{K}{6} > 1$

$\Rightarrow \boxed{K > 6}$

Q. Plot the Polar plot of system having open loop transfer function $G(s)H(s) = \frac{K}{s(1+T_1 s)(1+T_2 s)}$ and determine its stability using concept of enclosement.

Soln:- Given OLTF of system

$$G(s)H(s) = \frac{K}{s(1+T_1 s)(1+T_2 s)}$$

$$= K\, G_1(s)H_1(s)$$

Let us draw the polar plot of $G_1(s)H_1(s)$.

$$\therefore G_1(s)H_1(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)} \quad \text{(time constant form)}$$

Put $s = j\omega$

$$G_1(j\omega)H_1(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

$$|G_1(j\omega)H_1(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}}$$

$$\angle G_1(j\omega)H_1(j\omega) = -90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

Also: $$G_1(j\omega)H_1(j\omega) = \frac{1(0-j\omega)(1-j\omega T_1)(1-j\omega T_2)}{(0^2+\omega^2)(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \frac{(-j\omega)(1-j\omega T_1-j\omega T_2-\omega^2T_1T_2)}{\omega^2(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \frac{-\omega(T_1+T_2) - j(1-\omega^2T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$= \frac{-\omega(T_1+T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)} - j\frac{(1-\omega^2T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$\therefore \; Re\left[G_1(j\omega)H_1(j\omega)\right] = \frac{-\cancel{\omega}(T_1+T_2)}{\cancel{\omega}(1+\omega^2T_1^2)(1+\omega^2T_2^2)} = \frac{-(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

$$Im\left[G_1(j\omega)H_1(j\omega)\right] = \frac{-(1-\omega^2T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$$

ω	$\lvert G_1(j\omega)H_1(j\omega)\rvert = \frac{1}{\omega\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}}$	$\angle G_1(j\omega)H_1(j\omega) = -90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$	$Re\left[G_1(j\omega)H_1(j\omega)\right] = \frac{-(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$	$Im\left[G_1(j\omega)H_1(j\omega)\right] = \frac{-(1-\omega^2T_1T_2)}{\omega(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$
0	∞	-90°	$-(T_1+T_2)$	$-\infty$
∞	0	-270°	-0	$+0$

Intersection of Polar plot with Real axis.

$$Im\left[G_1(j\omega)H_1(j\omega)\right] = 0$$

$$\Rightarrow 1-\omega^2T_1T_2 = 0$$

$$\Rightarrow \omega^2T_1T_2 = 1$$

$$\Rightarrow \boxed{\omega = \frac{1}{\sqrt{T_1T_2}}}$$

$$Re\left[G_1(j\omega)H_1(j\omega)\right]\Bigg|_{\omega=\frac{1}{\sqrt{T_1T_2}}}$$

$$= \frac{-(T_1+T_2)}{\left(1+\frac{T_1^2}{T_1T_2}\right)\left(1+\frac{T_2^2}{T_1T_2}\right)}$$

$$= \frac{-(T_1+T_2)}{\left(1+\frac{T_1}{T_2}\right)\left(1+\frac{T_2}{T_1}\right)}$$

$$= \frac{-(T_1+T_2)}{\frac{(T_1+T_2)^2}{T_1T_2}} = -\left(\frac{T_1T_2}{T_1+T_2}\right)$$

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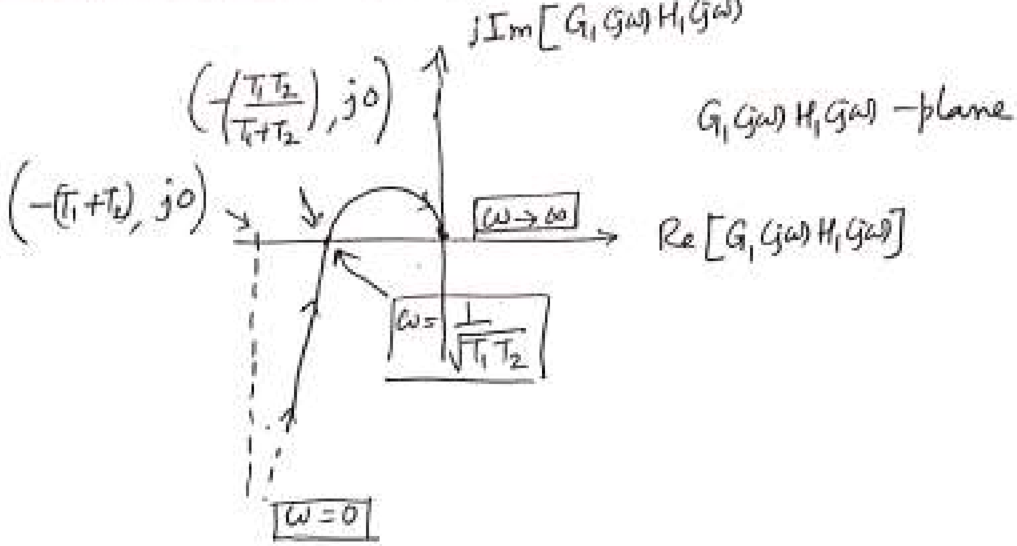
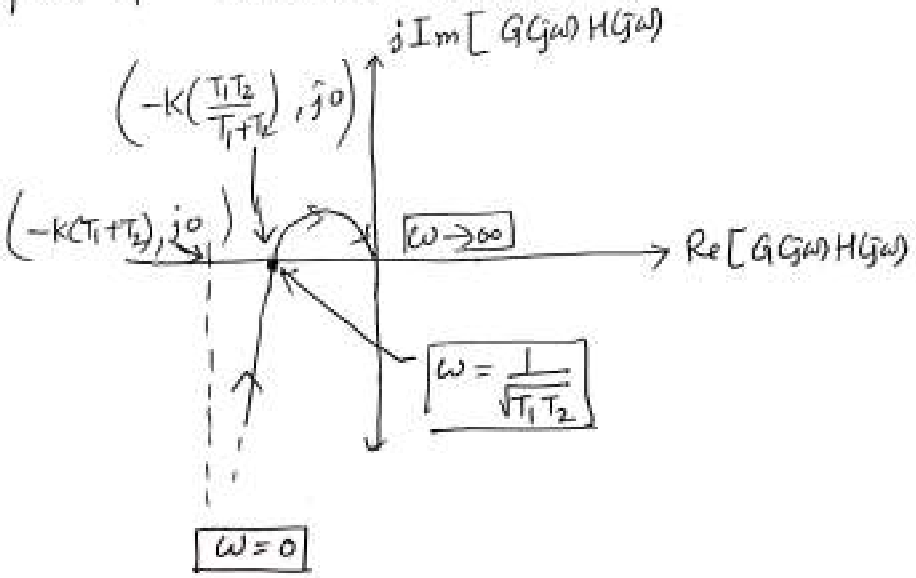
Polar plot of $G_1(s)H_1(s)$



Polar plot of $G(s)H(s) = K\,G_1(s)H_1(s)$



Closed loop stability of System :-

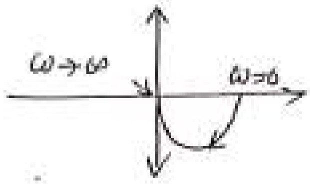
Absolute Stable	Marginally Stable	Unstable
→ Critical point shouldn't be enclosed	→ Critical point should lie on the plot	→ Critical point should be enclosed
→ Critical point $(-1, j0)$ should lie to the LHS of plot	→ $(-1, j0)$ should lie on the plot	→ $(-1, j0)$ should lie to the RHS of plot
$-K\left(\frac{T_1T_2}{T_1+T_2}\right) > -1$	$-\left(\frac{KT_1T_2}{T_1+T_2}\right) = -1$	$-K\left(\frac{T_1T_2}{T_1+T_2}\right) < -1$
$\Rightarrow \boxed{K < \frac{T_1+T_2}{T_1T_2}}$	$\Rightarrow \boxed{K = \frac{T_1+T_2}{T_1T_2}}$	$\Rightarrow \boxed{K > \frac{T_1+T_2}{T_1T_2}}$

Some standard Polar plots :-

OLTF — **Polar plot**

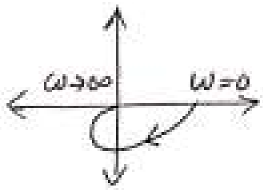
1. $G(s)H(s) = \dfrac{1}{(1+Ts)}$

Type → 0, Order → 1



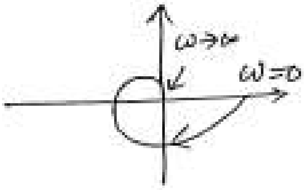
2. $G(s)H(s) = \dfrac{1}{(1+T_1 s)(1+T_2 s)}$

Type → 0, Order → 2



3. $G(s)H(s) = \dfrac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$

Type → 0, Order → 3



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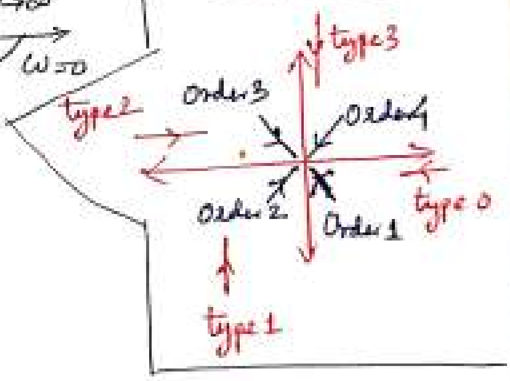
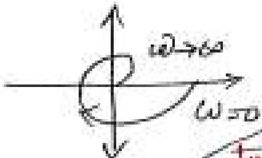
General



4. $G(s)H(s) = \dfrac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)}$

Type → 0, Order → 4



5. $G(s)H(s) = \dfrac{1}{s}$

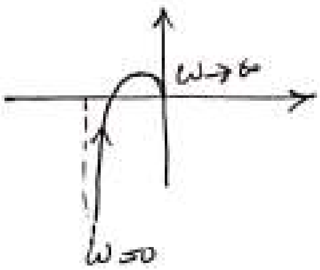
Type → 1, Order - 1

6. $G(s)H(s) = \dfrac{1}{s(1+Ts)}$

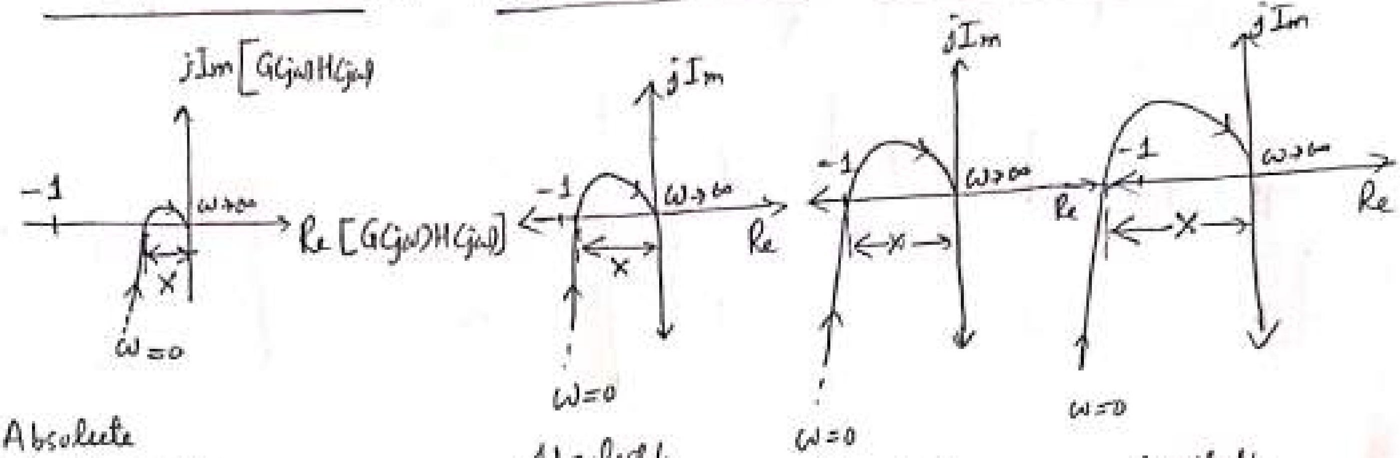
Type-1, Order:2

7. $G(s)H(s) = \dfrac{1}{s(1+T_1 s)(1+T_2 s)}$

Type-1, Order → 3



RELATIVE STABILITY : GAIN MARGIN (GM) & PHASE MARGIN (PM).



Absolute stable system (A) | Absolutely stable system (B) | Marginally stable system (C) | unstable system. (D)

Closed loop stability	Critical point	X
A → Absolute stable	not enclosed	0.5 (< 1)
B → Absolute stable	not enclosed	0.75 (< 1)
C → Marginally stable	Not enclosed; but lies on the polar plot	= 1 (> 1)
D → Unstable	enclosed	1.25 (> 1)

Between A & B, System A is more stable than system B.

Hence $\boxed{\text{stability} \propto \frac{1}{X}}$

And $\boxed{\frac{1}{X} = \text{Gain Margin}}$

in dB ⇛

$$\boxed{\begin{aligned} GM(dB) &= 20\log_{10}(GM) \\ GM(dB) &= 20\log_{10}\left(\frac{1}{X}\right) \end{aligned}}$$

	X	$GM = \frac{1}{X}$	$GM(dB) = 20\log_{10}(GM)$	PM	ω_{pc} ? ω_{gc}
Absolute Stable	$X < 1$	> 1	+ve	+ve	$\omega_{pc} > \omega_{gc}$
Marginally Stable	$X = 1$	$= 1$	0	0	$\omega_{pc} = \omega_{gc}$
Unstable	$X > 1$	< 1	−ve	−ve	$\omega_{pc} < \omega_{gc}$

Gain Margin (GM) : Phase Cross-Over :- A phase cross-over on the Nyquist plot is a point at which the plot intersects the −ve Real axis.

Phase Cross Over frequency (ω_{pc}) : The frequency at which phase cross-over or

$\angle G(j\omega)H(j\omega) = -180^\circ$ is called the phase-crossover frequency.

Gain Margin of the closed loop system having $G(s)H(s)$ as loop transfer function is defined as :-

$$\text{Gain Margin} = GM(dB) = 20\log_{10} \left. \frac{1}{|G(j\omega)H(j\omega)|} \right|_{\omega = \omega_{pc}} (dB)$$

$$= -20\log_{10} \left. |G(j\omega)H(j\omega)| \right|_{\omega = \omega_{pc}} (dB)$$

Physical Significance of Gain Margin

Gain Margin is the amount of gain in decibels (dB) that can be added to the loop before the closed-loop system becomes unstable.

NOTE :-

→ When Nyquist plot doesnot intersect the negative real axis at any finite non-zero frequency, the Gain Margin in dB infinite (∞) ; this means that, theoritically, the value of the loop gain can be increased to infinity before the system becomes unstable

→ When Nyquist plot passes through $(-1, j0)$, $GM(dB) = 0dB$. which implies that loop gain can nolonger be increased, because the system is at the margin of instability

→ When the phase crossover is to the left of critical point $(X > 1)$, the $GM(dB)$ value is -ve, and the loop Gain must be reduced by $GM(dB)$ to acheive stability.

Imp ※ ~~These are valid~~ for ~~All minimum-phase loop transfer function~~

※ But even for non-minimum phase system, the closeness of phase cross-over to $(-1, j0)$ still gives an indication of relative stability.

Phase Margin (PM) :-

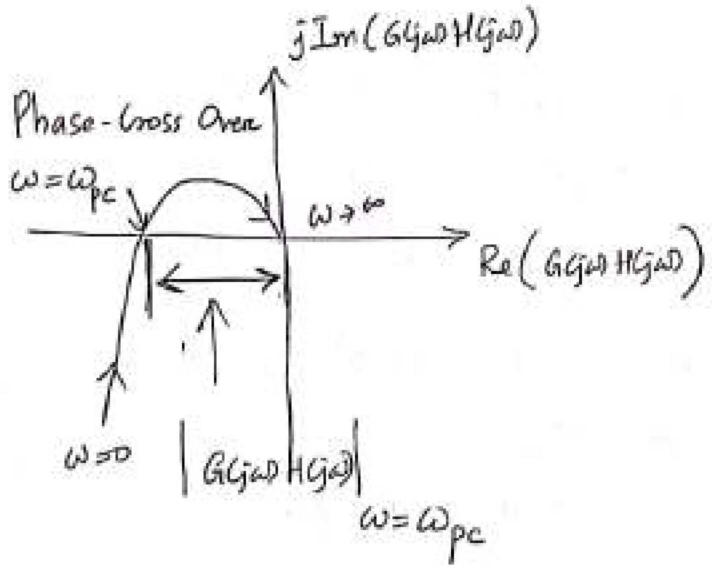
Gain Cross-Over :- Gain cross-over is a point on the Nyquist plot at which $|G(j\omega)H(j\omega)| = 1$

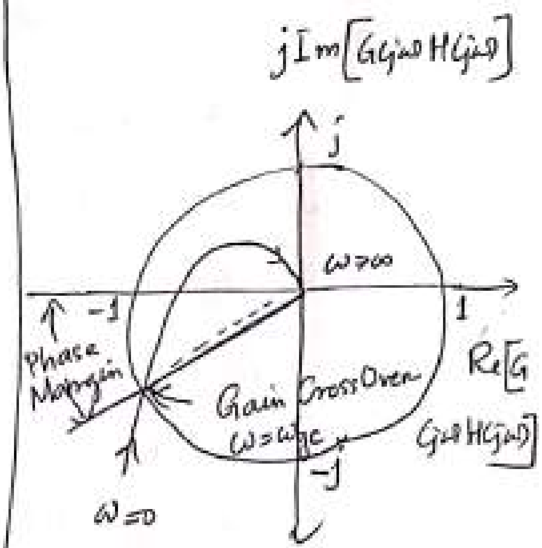
Gain Cross-Over Frequency (ω_{gc}) :- Gain Cross-Over frequency is the frequency at which the gain crossover. or where $|G(j\omega)H(j\omega)| = 1$

$$\phi = \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}}$$

$$\text{Phase Margin (PM)} = 180^\circ - |\phi|$$

Phase Margin is defined as the angle in degrees through which the Nyquist plot must be rotated about the origin so that the gain crossover passes through the (-1, j0) point.





$$GM(dB) = 20\log_{10}\frac{1}{|G(j\omega)H(j\omega)|}\Bigg|_{\omega=\omega_{pc}} \text{ dB}$$

$$= -20\log_{10}|G(j\omega)H(j\omega)|\Big|_{\omega=\omega_{pc}} \text{ dB}$$

MINIMUM-PHASE TRANSFER FUNCTION :-

1. A minimum phase transfer function doesnot have poles or zeros in the right half s-plane, or on the $j\omega$-axis, excluding the origin.
2. For a minimum phase transfer function $L(s)$ with m-zeros and n-poles, excluding the poles at $s=0$, when $s=j\omega$ and as ω varies from 0 to ∞, the total phase variation of $L(j\omega)$ is $(n-m)\frac{\pi}{2}$ radians.
3. The value of minimum phase transfer function cannot become zero or ∞ at any finite nonzero frequency.

Q) Given $G(s) = \frac{2\sqrt{3}}{s(s+1)}$, $H(s) = 1$

Determine GM and PM and comment on closed loop stability of the system.

Solution :- Given, $G(s)H(s) = \frac{2\sqrt{3}}{s(s+1)}$

Poles are at $s=0$, $s=-1$

It is a minimum phase transfer function.

Put $s = j\omega$,

$$G(j\omega)H(j\omega) = \frac{2\sqrt{3}}{j\omega\,(1+j\omega)}$$

$$|G(j\omega)H(j\omega)| = \frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = 0^\circ - 90^\circ - \tan^{-1}(\omega) = -90^\circ - \tan^{-1}(\omega).$$

Gain Margin :-

Phase Crossover Frequency $\omega_{pc} = ?$

At phase-crossover,

$$\angle G(j\omega)H(j\omega) = -180^\circ$$

$$\Rightarrow -90^\circ - \tan^{-1}(\omega) = -180^\circ$$

$$\Rightarrow \tan^{-1}(\omega) = 90^\circ$$

$$\Rightarrow \boxed{\omega_{pc} = \infty}$$

$$\therefore x = |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = \left.\frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}}\right|_{\omega=\omega_{pc}=\infty} = 0$$

$$GM = \frac{1}{x} = \frac{1}{0} = \infty$$

$$GM\,(dB) = 20\log_{10}(GM)$$

$$= 20\log_{10}(\infty) = \infty \text{ dB}.$$

PHASE MARGIN (PM)

Gain Cross Over frequency: (ω_{gc})

At gain cross-over,

$$|G(j\omega)H(j\omega)| = 1$$

$$\Rightarrow \frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}} = 1$$

$$\Rightarrow 2\sqrt{3} = \omega\sqrt{1+\omega^2}$$

By observation $\omega^2 = 3$

$$\Rightarrow \boxed{\omega = \sqrt{3}}$$

$$\therefore \boxed{\omega_{gc} = \sqrt{3}}$$

$$\phi = \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}=\sqrt{3}}$$

$$= -90^\circ - \tan^{-1}(\omega)\Big|_{\omega_{gc}=\sqrt{3}}$$

$$= -90^\circ - \tan^{-1}(\sqrt{3})$$

$$= -90^\circ - 60^\circ = -150^\circ$$

$$\therefore PM = 180 - |\phi|$$

$$= 180^\circ - |-150^\circ|$$

$$= 180^\circ - 150^\circ$$

$$\Rightarrow \boxed{PM = 30^\circ}$$

<u>Closed loop stability</u>

Here $\omega_{pc} = \infty$, $GM(dB) = \infty$ dB, $\omega_{gc} = \sqrt{3}$ rad/sec, $PM = 30^\circ$

$GM = \infty$

Since $GM(dB) > 0$ and $PM > 0$ and $\omega_{pc} > \omega_{gc}$, the system is absolute stable.

<u>Q</u> A unity feedback control system has $G(s) = \dfrac{80}{s(s+2)(s+20)}$

Determine GM, PM, ω_{pc}, ω_{gc}. Also comment on its stability.

<u>Solution</u>: GM, PM, ω_{pc} & ω_{gc} can be evaluated in 2 methods:-

(1) From Bode Plot

(2) Mathematically (Nyquist plot)

We will chose the 2^{nd} approach here.

Given, $G(s)H(s) = \dfrac{80}{s(s+2)(s+20)} = L(s)$ [say]

$$\therefore L(s) = \frac{80}{s \times 2\left(1 + \frac{1}{2}s\right) \times 20\left(1 + \frac{1}{20}s\right)}$$

$$= \frac{\frac{80}{2 \times 20}}{s\left(1 + \frac{1}{2}s\right)\left(1 + \frac{1}{20}s\right)} = \frac{2}{s\left(1 + \frac{1}{2}s\right)\left(1 + \frac{1}{20}s\right)}$$

(time constant form)

Put $s = j\omega$

$$L(j\omega) = \frac{2}{j\omega(1+j\frac{1}{2}\omega)(1+j\frac{1}{20}\omega)}$$

$$|L(j\omega)| = \frac{2}{\omega\sqrt{\left(1+\frac{\omega^2}{4}\right)\left(1+\frac{\omega^2}{400}\right)}}$$

$$\angle L(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$$

Gain Margin (GM) :-

Phase CrossOver frequency : (ω_{pc})

At phase-cross over

$$\angle L(j\omega) = -180^\circ$$

$$\Rightarrow -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right) = -180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{20}\right) = 90^\circ$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{\omega}{2}+\frac{\omega}{20}}{1-\frac{\omega}{2}\cdot\frac{\omega}{20}}\right] = 90^\circ$$

$$\Rightarrow \frac{\frac{\omega}{2}+\frac{\omega}{20}}{1-\frac{\omega^2}{40}} = \infty$$

$$\Rightarrow 1 - \frac{\omega^2}{40} = 0$$

$$\Rightarrow \frac{\omega^2}{40} = 1$$

$$\Rightarrow \omega = \sqrt{40} \text{ rad/sec}$$

$$\therefore \boxed{\omega_{pc} = \sqrt{40} \text{ rad/sec}}$$

$$X = |G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}$$

$$\therefore GM = \frac{1}{X}$$

$$= \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} \quad \text{or} \quad \frac{1}{|L(j\omega)|_{\omega=\omega_{pc}}}$$

$$= \left. \frac{\omega\sqrt{\left(1+\frac{\omega^2}{4}\right)\left(1+\frac{\omega^2}{400}\right)}}{2} \right|_{\omega=\omega_{pc}=\sqrt{40}}$$

$$= \sqrt{40}\sqrt{\left(1+\frac{40}{4}\right)\left(1+\frac{40}{400}\right)}$$

$$= \sqrt{\cancel{40}^{4} \times 11 \times \frac{11}{\cancel{10}}}$$

$$\boxed{GM = 22}$$

$$GM(dB) = 20\log_{10}(GM)$$

$$= 20\log_{10}(22)$$

$$\boxed{GM(dB) = 26.85\ dB}$$

Phase Margin (PM)

Gain Cross Over Frequency :- (ω_{gc})

At Gain crossover:

$$|G(j\omega)H(j\omega)| = |L(j\omega)| = 1$$

$$\Rightarrow \frac{2}{\omega\sqrt{\left(1+\frac{\omega^2}{4}\right)\left(1+\frac{\omega^2}{400}\right)}} = 1$$

$$\Rightarrow \omega\sqrt{\left(1+\frac{\omega^2}{4}\right)\left(1+\frac{\omega^2}{400}\right)} = 2$$

$$\Rightarrow \omega^2\left(1+\frac{\omega^2}{4}\right)\left(1+\frac{\omega^2}{400}\right) = 4$$

$$\Rightarrow \frac{\omega^2(4+\omega^2)(400+\omega^2)}{4\times 400} = 4$$

$$\Rightarrow \omega^2(4+\omega^2)(400+\omega^2) = 4\times 4\times 400$$

$$\Rightarrow \omega^2(\omega^4 + 404\omega^2 + 1600) = 6400$$

$$\Rightarrow \omega^6 + 404\omega^4 + 1600\omega^2 - 6400 = 0$$

Let $\omega^2 = x$

$$\therefore x^3 + 404x^2 + 1600x - 6400 = 0$$

$$\Rightarrow x = 2.46,\ -399.96,\ -6.50$$

Consider only $x = 2.46$

$$\Rightarrow \omega^2 = 2.46$$

$$\Rightarrow \omega = \sqrt{2.46} = 1.57 \simeq \frac{\pi}{2}\ \text{rad/sec}$$

$$\therefore \boxed{\omega_{gc} = \frac{\pi}{2}\ \text{rad/sec}}$$

$$\phi = \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}} = \angle L(j\omega)\Big|_{\omega=\omega_{gc}} = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)\Big|_{\omega=\omega_{gc}}$$

$$= -90^\circ - \tan^{-1}\left(\frac{\pi}{4}\right) - \tan^{-1}\left(\frac{\pi}{40}\right)$$

$$= -90^\circ - 38.15^\circ - 4.5^\circ$$

$$= -132.65^\circ$$

$$\therefore PM = 180^\circ - |\phi|$$
$$= 180^\circ - |-132.65^\circ|$$
$$= 180^\circ - 132.65^\circ$$
$$\boxed{PM = 47.35^\circ}$$

Here

$\omega_{pc} = \sqrt{40}$ $= 6.32$ rad/sec	$GM(dB) = 26.85$ dB $GM = 22$	$\omega_{gc} = 1.57$ rad/sec	$PM = 47.35^\circ$

The system is absolutely stable as :-

$$\left.\begin{array}{l} GM(dB) > 0 \\ PM > 0 \\ \text{and } \omega_{pc} > \omega_{gc} \end{array}\right\}.$$

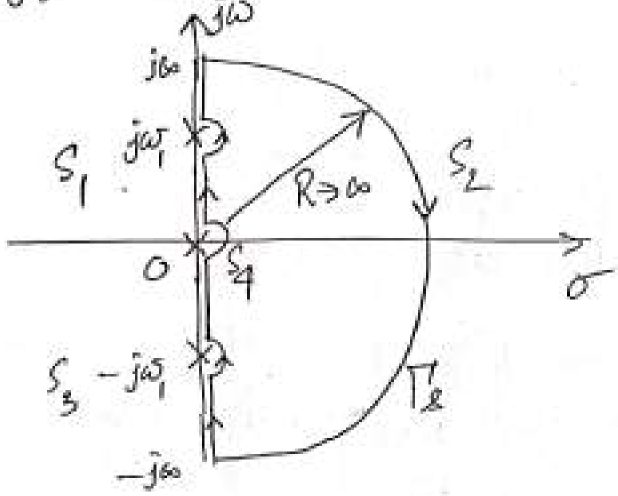
(1)

Nyquist Plot

$\boxed{\omega : -\infty \text{ to } +\infty}$

NYQUIST CONTOUR :-

or (Nyquist path)



Characteristics equation,

$\Delta(s) = 1 + G(s)H(s) = 0$

$\Rightarrow \Delta(s) = 1 + L(s) = 0$

Nyquist path does not pass through any poles or zeros of $\Delta(s)$ on $j\omega$ axis

Regions of Nyquist path

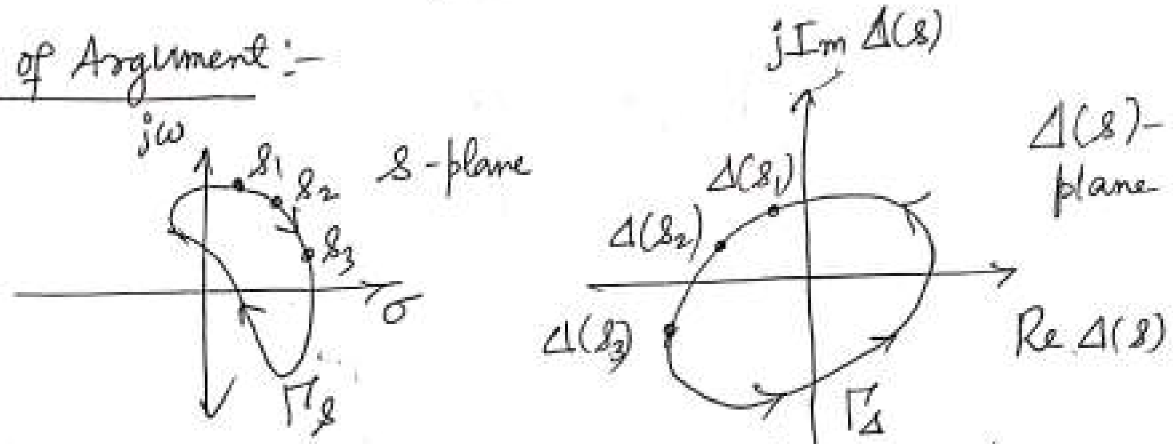
S_1: $s = j\omega$, ω varies from $0+$ to ∞

S_2: $s = \lim_{R\to\infty} Re^{j\theta}$, $\theta \to \frac{\pi}{2}$ to $-\frac{\pi}{2}$

S_3 :- $s = j\omega$, ω varies from $-\infty$ to 0^-

S_4: $s = \lim_{R\to 0} Re^{j\theta}$, $\theta \to -\frac{\pi}{2}$ to $\frac{\pi}{2}$

Principle of Argument :-



→ Let Δ(s) be a single valued function of the form of :

$$\frac{K(1+T_1 s)(1+T_2 s)\cdots(1+T_m s)}{s^p(1+T_a s)(1+T_b s)\cdots(1+T_n s)}e^{-T_d s}$$

which has a finite number of poles in the s-plane.

→ Single valued means that for each point in the s-plane there is one and only one corresponding point, including infinity in the complex Δ(s) plane.

→ If Γ_s doesnot go through any poles of Δ(s), then the trajectory Γ_Δ mapped by Δ(s) into Δ(s)-plane is also a closed one.

→ The principle of argument can be stated as :-

Let Δ(s) be a single valued function that has a finite number of poles in s-plane. Suppose that an arbitrary closed path Γ_s is chosen in the s-plane so that the path doesnot go through any one of the poles or zeros of Δ(s), the corresponding Γ_Δ locus mapped in the Δ(s)-plane will encircle the origin as many times as the difference between the number of zeros and poles of Δ(s) that are encircled by the s-plane locus Γ_s.

In equation form principle of argement is :-

$$\boxed{N = P - Z}$$

where

N = number of encirclement of the origin made by $\Delta(s)$-plane Locus Γ_Δ

(or) The number of encirclements of Critical point $(-1, j0)$ made by Nyquist plot

N is positive for counter clock wise (CCW) encirclement & N is negative for clock wise (CW) encirclement.

Z = No of closed loop poles in Right Half s-plane
= No of open loop zeros in " " "

P = No of open loop poles in Right half of s-plane

For system to be closed loop stable

$$Z = 0$$

$$\Rightarrow \boxed{N = P}$$

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Q) Draw the Nyquist plot of a system having open loop transfer function, $G(s)H(s) = \frac{10}{(s+2)(s+1)}$

And determine system stability.

Soln:- Given system $G(s)H(s) = \frac{10}{(s+2)(s+1)}$

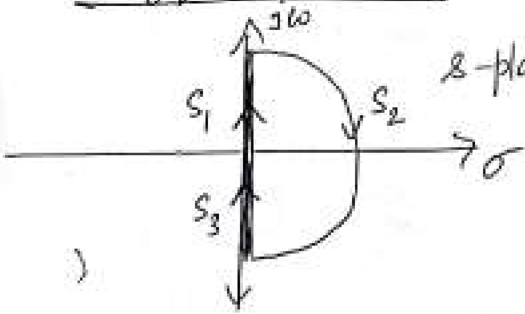
Type → 0, Order → 2

open loop poles are at $s = -1, -2$

⇒ No of open loop finite poles to the Right Half s-plane,

$\boxed{P = 0}$

Nyquist Contour

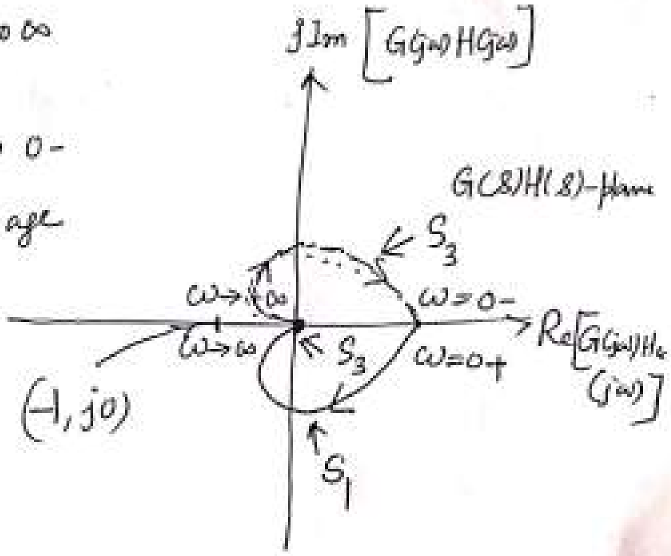


Region: S_1: ω varies from 0+ to ∞

It gives the polar plot or Nyquist plot for ω = 0+ to ∞

Region S_3: ω varies from −∞ to 0−

It gives the mirror image of polar plot w.r.t real axis.



Region S_2: $s = \lim_{R\to\infty} Re^{j\theta}$

$\theta \to \frac{\pi}{2}$ to $-\frac{\pi}{2}$

$$\therefore \frac{10}{(s+2)(s+1)} \simeq \frac{10}{s \cdot s} = \frac{10}{s^2} = \frac{10}{\lim_{\substack{R\to\infty \\ \theta \to +\pi/2 \text{ to } -\pi/2}} (Re^{j\theta})^2}$$

$$= \frac{10}{\lim\limits_{\substack{R\to\infty \\ \theta \to \pi/2 \text{ to } -\pi/2}} R^2 e^{j2\theta}} = \frac{10}{\infty^2} e^{-j2\left(+\pi/2 \text{ to } -\pi/2\right)}$$

$$= 0\, e^{j[-\pi \text{ to } +\pi]}$$

No. of encirclement of critical point by Nyquist plot is:-

$$\boxed{N = 0}$$

$$\therefore \quad N = P - Z$$
$$\Rightarrow Z = P - N$$
$$\Rightarrow Z = 0 - 0$$
$$\Rightarrow \boxed{Z = 0}$$

No of closed loop poles to the Right of s-plane is 0 so the system is absolutely stable.

Q) 2017

5(c) By using Nyquist criteria determine whether the closed loop system is stable or not for given open loop T/F

$$G(s)H(s) = \frac{1}{s(1+2s)(1+s)}$$

Soln:- Steps:- (I) Draw Polar plot $(\omega: 0+ \to \infty)$

(II) Draw Nyquist plot $(\omega: -\infty \text{ to } +\infty)$

(III) Apply Principle of argument to determine closed loop stability of system.

POLAR PLOT $G(s)H(s) = \dfrac{1}{s(1+s)(1+2s)}$ [in time constant form & $K=1$]

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+j\omega)(1+j2\omega)}$$

$\boxed{\begin{array}{c}\text{Type} \to 1 \\ \text{Order}-3\end{array}}$

$$|G(j\omega)H(j\omega)| = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}}$$

$$\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

Also $G(j\omega)H(j\omega) = \dfrac{1}{j\omega(1+j\omega)(1+j2\omega)}$

$$= \frac{-j(1-j\omega)(1-j2\omega)}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-j\left[1-2\omega^2-j3\omega\right]}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-3\omega}{\omega(1+\omega^2)(1+4\omega^2)} - j\frac{(1-2\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-3}{(1+\omega^2)(1+4\omega^2)} - j\frac{(1-2\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$$

$Re[G(j\omega)H(j\omega)] = \dfrac{-3}{(1+\omega^2)(1+4\omega^2)}$, $Im[G(j\omega)H(j\omega)] = \dfrac{-(1-2\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$

Intersection with Real axis

On Real axis

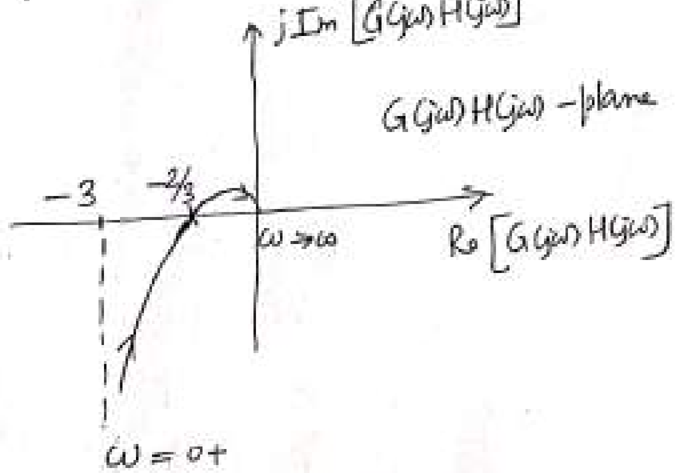
$$Im[G(j\omega)H(j\omega)] = 0$$

$$\Rightarrow 1-2\omega^2 = 0$$

$$\Rightarrow \omega^2 = \frac{1}{2} \Rightarrow \omega = \frac{1}{\sqrt{2}}$$

$$\therefore Re[G(j\omega)H(j\omega)]\Big|_{\omega=\frac{1}{\sqrt{2}}} = \frac{-3}{(1+\frac{1}{2})(1+\frac{4}{2})} = -\frac{2}{3}$$

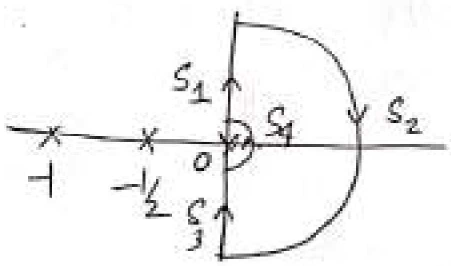
ω	$\lvert G(j\omega)H(j\omega)\rvert = \frac{1}{\omega\sqrt{(1+\omega^2)(1+4\omega^2)}}$	$\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$	$Re[G(j\omega)H(j\omega)] = \frac{-3}{(1+\omega^2)(1+4\omega^2)}$	$Im[G(j\omega)H(j\omega)] = \frac{-(1-2\omega^2)}{\omega(1+\omega^2)(1+4\omega^2)}$
0	∞	-90°	-3	$-\infty$
∞	0	-270°	-0	$+0$



<u>Nyquist Contour</u> : $G(s)H(s) = \frac{1}{s(1+s)(1+2s)}$

Poles : $s = 0, -1, -\frac{1}{2}$

$\boxed{P = 0}$



<u>Region S_1</u> :- ω varies from $0+$ to ∞

It gives the polar plot

<u>Region S_3</u> :- ω varies from $-\infty$ to $0-$

It gives the mirror image of polar plot about Real axis.

Region S_2 :-

$$s = \lim_{\substack{R\to\infty \\ \theta \to +\frac{\pi}{2} \text{ to } -\frac{\pi}{2}}} Re^{j\theta}$$

$$\therefore \frac{1}{s(1+s)(1+2s)} \simeq \frac{1}{s^3} = \frac{1}{\lim\limits_{\substack{R\to\infty \\ \theta\to +\frac{\pi}{2} \text{ to } -\frac{\pi}{2}}} (Re^{j\theta})^3} = \frac{1}{(\infty)^3} e^{-j\left[\frac{3\pi}{2} \text{ to } -\frac{3\pi}{2}\right]}$$

$$= 0 \; e^{+j\left[\frac{-3\pi}{2} \text{ to } \frac{3\pi}{2}\right]}$$

Region S_4

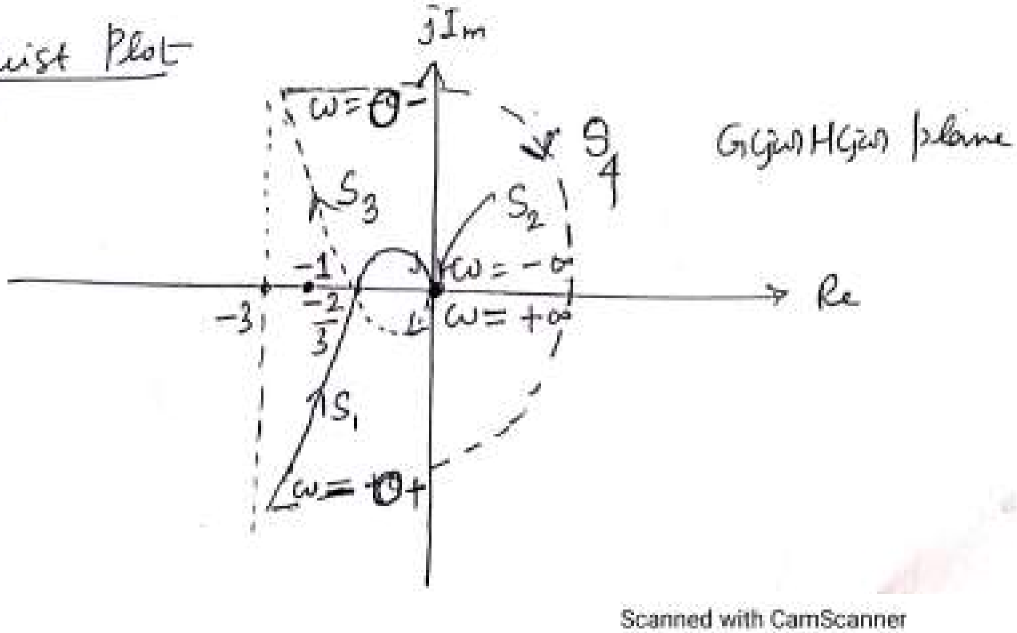
$$s = \lim_{\substack{R\to 0 \\ \theta \to -\frac{\pi}{2} \text{ to } \frac{\pi}{2}}} Re^{j\theta}$$

$$\frac{1}{s(1+s)(1+2s)} \simeq \frac{1/2}{s(s+1)(s+\frac{1}{2})} \simeq \frac{1/2}{s \times 1 \times \frac{1}{2}}$$

$$= \frac{1}{s} = \frac{1}{\lim\limits_{\substack{R\to 0 \\ \theta\to -\frac{\pi}{2} \text{ to } \frac{\pi}{2}}} Re^{j\theta}} = \frac{1}{0} e^{-j\left(-\frac{\pi}{2} \text{ to } \frac{\pi}{2}\right)}$$

$$= \infty \; e^{j\left(\frac{\pi}{2} \text{ to } -\frac{\pi}{2}\right)}$$

Nyquist Plot



No of encirclements of critical point

$\boxed{N=0}$

$\boxed{P=0}$, Given

$N = P - Z$

$\Rightarrow Z = P - N$

$\Rightarrow Z = 0 - 0$

$\Rightarrow \boxed{Z = 0}$

No of closed loop poles to the Right half s-plane = 0

$\Rightarrow$ System is closed loop stable.

①

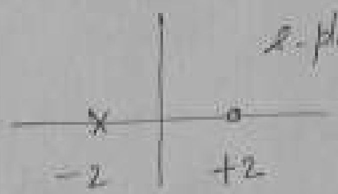
All pass System :-

(i) An all pass system has magnitude of its transfer function equal to constant for all frequency.

ie, $|H(j\omega)| = \text{constant} = K \quad \forall \omega$

(ii) For an all pass system poles must lie on the left and zeros on the mirror image of the poles can be on the right.

$$H(s) = \frac{s-2}{s+2}$$



Put $s = j\omega$

$$H(j\omega) = \frac{j\omega - 2}{j\omega + 2}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 2^2}}{\sqrt{\omega^2 + 2^2}} = 1, \quad \forall \omega$$

Advantages of Nyquist Plot :

① The Nyquist plot can be used for study of stability of system with non-minimum phase transfer function.

② The stability analysis of a closed loop system can be easily investigated by examining the Nyquist plot of the loop transfer function with reference to the $(-1, j0)$ point.

DISADVANTAGE OF NYQUIST PLOT :

It is not so easy to carry out the design of controllers by referring to the Nyquist plot.

BODE PLOT : (Asymptotic Plot)

ADVANTAGES :-

1. In the absence of a computer, A Bode diagram can be sketched by approximating the magnitude and Phase with straight line segment.
2. Gain Cross Over, Phase Cross Over, Gain Margin, Phase Margin, are more easily determined on the Bode plot rather than from the Nyquist plot.
3. For design purpose the effect of adding controllers and their parameters are more easily acheived/vizualized on the Bode plot than on Nyquist plot.

DISADVANTAGES :

1. Absolute stability & relative stability of only minimum phase system can be acheived from Bode plot. There is no way of telling what the stability criterion is on the Bode plot.

→ Bode plot is a plot of the dB value of the magnitude and phase angle in degrees vs. $\log\omega$.

Q) Given $G(s) = \dfrac{10^3 (s+20)}{(s+10)(s+200)}$

Draw the Bode plot.

Determine ω_{pc}, ω_{gc}, GM and PM from the plot.

Soln: Given $G(s) = \dfrac{10^3 (s+20)}{(s+10)(s+200)}$

Writing it in time constant form.

$$G(s) = \frac{10^3 \times 20 \left(1 + \frac{1}{20}s\right)}{10 \times 200 \times \left(1 + \frac{1}{10}s\right)\left(1 + \frac{1}{200}s\right)}$$

$$= \frac{10\left(1 + \frac{s}{20}\right)}{\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{200}\right)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{10\left(1 + j\frac{\omega}{20}\right)}{\left(1 + j\frac{\omega}{10}\right)\left(1 + j\frac{\omega}{200}\right)}$$

(i) $K = 10$

$20\log_{10} K = 20\log_{10} 10 = 20\,\text{dB}$

(ii) Here no $(j\omega)$ factor is present
So initial slope = 0

(iii) First Corner Frequency

factor: $\dfrac{1}{\left(1+j\frac{\omega}{10}\right)^1}$

$\omega_1 = 10$ rad/sec $\log_{10}\omega_1 = \log_{10} 10 = 1$

Change in slope at $\omega_1 = 10$ rad/sec is -20 dB/decade

(iv) 2^{nd} Corner frequency

factor: $\left(1 + j\frac{\omega}{20}\right)$

$\omega_2 = 20$ rad/sec $\log_{10}\omega_2 = \log_{10} 20 = 1.3$

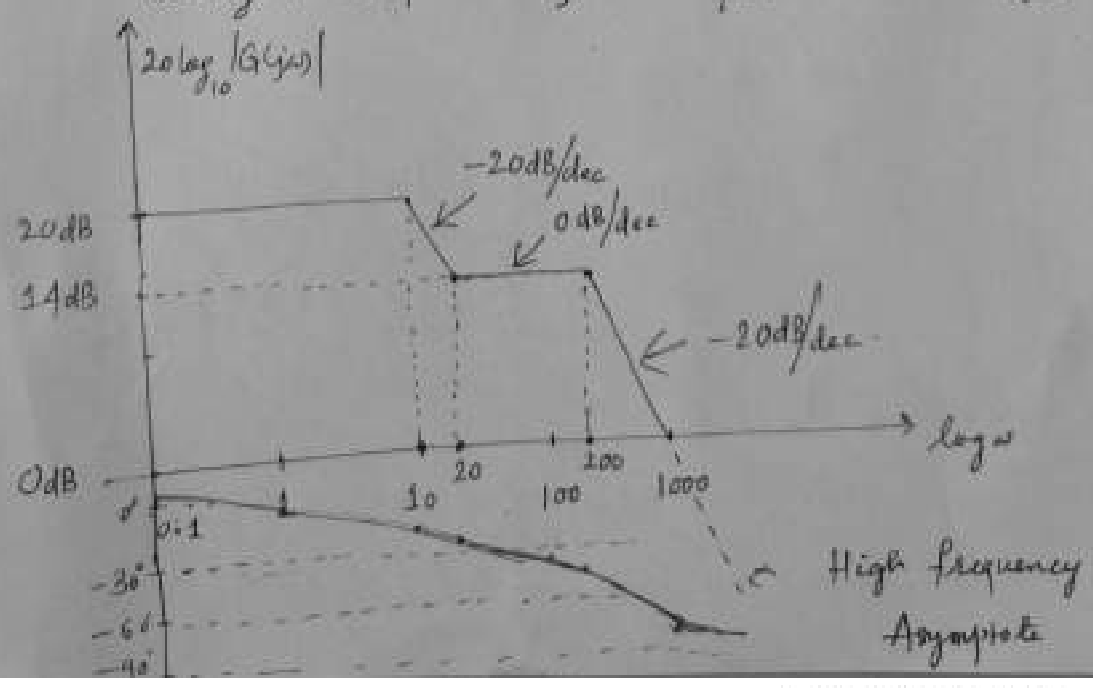
Change in slope at $\omega_2 = 20$ rad/sec is $+20$ dB/dec

(v) 3^{rd} Corner frequency

factor: $\dfrac{1}{\left(1+j\frac{\omega}{200}\right)}$

$\omega_3 = 200$ rad/sec, $\log_{10}\omega_3 = \log_{10} 200 = 2.3$

Change in slope at $\omega_3 = 200$ rad/sec is -20 dB/dec



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③

$$\angle G(j\omega) = \tan^{-1}(\omega/20) - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{200}\right)$$

ω	$-\tan^{-1}\left(\frac{\omega}{10}\right) + \tan^{-1}\left(\frac{\omega}{20}\right) - \tan^{-1}\left(\frac{\omega}{200}\right)$
0.1	$-0.57 + 0.29 - 0.03 = 0.31$
1	-3.13
10	-21.30
20	-24.14
100	-32.16
200	-47.85
1000	-79.26
∞	$-90^\circ + 90^\circ - 90^\circ = -90^\circ$

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M-Circles:- Consider that $G(s)$ is the forward path transfer function of a unity feedback system

The closed loop T/F is :-

$$M(s) = \frac{G(s)}{1+G(s)}$$

For sinusoidal steady state $s = j\omega$

$$\therefore\ G(j\omega) = Re[G(j\omega)] + j\,Im(G(j\omega))$$

$$= x + jy$$

where $x = Re[G(j\omega)]$, $y = Im[G(j\omega)]$

Magnitude of closed loop transfer function is :-

$$|M(j\omega)| = \left|\frac{G(j\omega)}{1+G(j\omega)}\right| = \left|\frac{x+jy}{1+x+jy}\right|$$

$$= \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

For simplicity, let M denote $|M(j\omega)|$

$$\therefore\ M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

$$\Rightarrow M\sqrt{(1+x)^2+y^2} = \sqrt{x^2+y^2}$$

$$\Rightarrow M^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2\left[1+x^2+2x+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2 + M^2x^2 + 2xM^2 + y^2M^2 = x^2+y^2$$

$$\Rightarrow (1-M^2)x^2 + (1-M^2)y^2 - 2xM^2 = M^2$$

$$\Rightarrow x^2+y^2 - \frac{2M^2}{1-M^2}x = \frac{M^2}{1-M^2}$$

$$\Rightarrow x^2 - 2\frac{M^2}{1-M^2}x + \left(\frac{M^2}{1-M^2}\right)^2 = \frac{M^2}{1-M^2} + \left(\frac{M^2}{1-M^2}\right)^2$$

$$\Rightarrow \left(x - \frac{M^2}{1-M^2}\right)^2 = \frac{M^2}{1-M^2}\left(1+\frac{M^2}{1-M^2}\right)$$

$$= \frac{M^2}{1-M^2}\left[\frac{1-M^2+M^2}{1-M^2}\right]$$

$$= \frac{M^2}{1-M^2} \times \frac{1}{1-M^2} = \left(\frac{M}{1-M^2}\right)^2$$

∴ Now,

$$\left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \left(\frac{M}{1-M^2}\right)^2, \quad M \neq 1$$

It represents a circle with

Centre at $\left(\frac{M^2}{1-M^2}, 0\right)$

& Radius $= \left|\frac{M}{1-M^2}\right|$.

When M takes different values the family of circles so formed are called constant-M loci or constant M circles.

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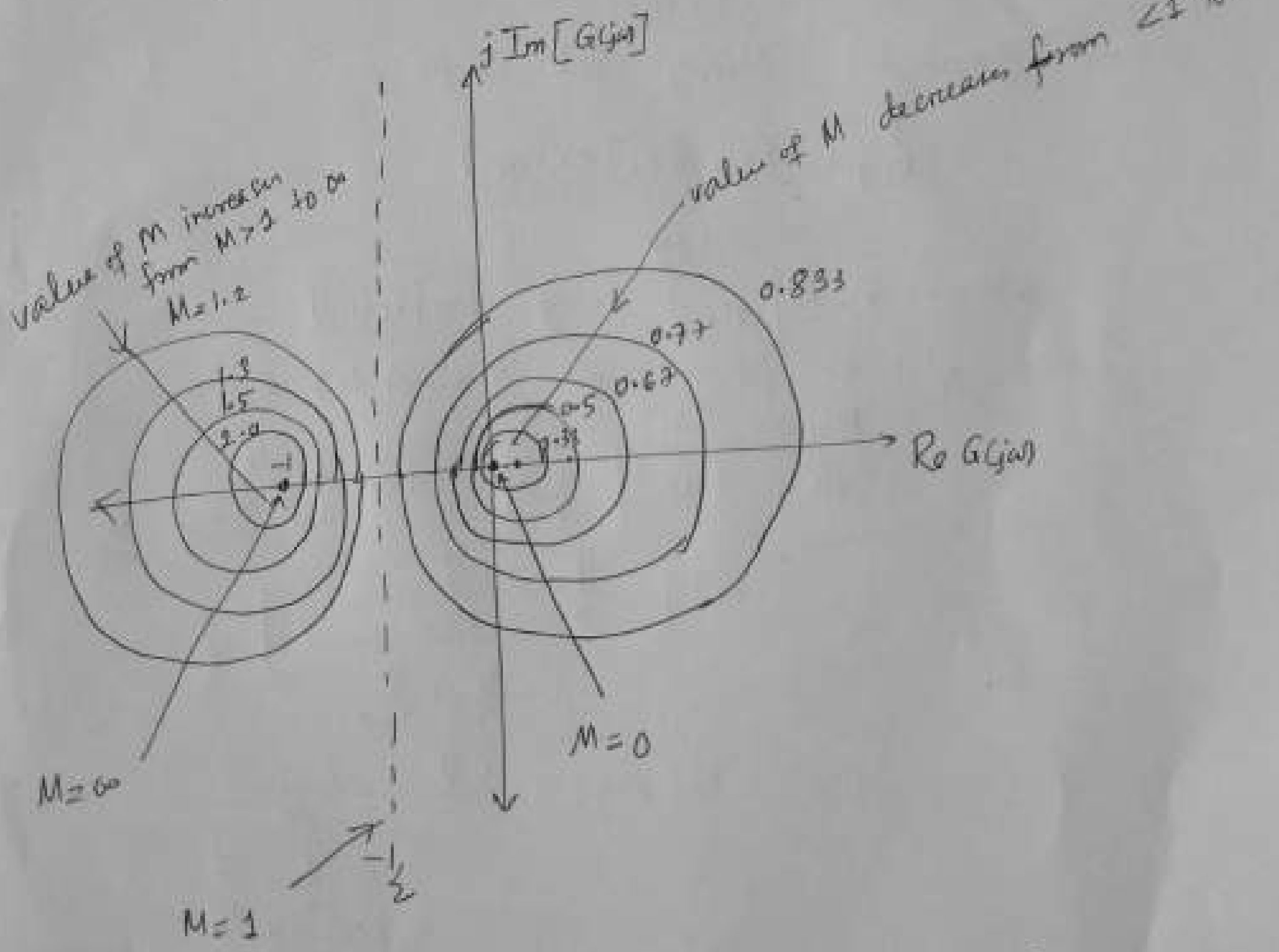
M	Centre $\left(\frac{M^2}{1-M^2}, 0\right)$	Radius $\left\lvert\frac{M}{1-M^2}\right\rvert$
0	(0,0)	0
0.5	(0.33,0)	0.66
$\neq$ 1	$\boxed{(\infty,0) \quad \infty}$	$\rightarrow x=-\frac{1}{2} \ \forall y$
2	(-1.33,0)	0.66
∞	(-1,0)	0



→ The circles are symmetrical w.r.t M=1 line and the real axis.

→ The circles to the left of M=1 locus corresponds to values of M greater than 1 and those to the right of the M=1 line are for M less than 1.

→ The constant M circle with the smallest radius that is tangent to the $G(j\omega)$ curve gives the value of M_r and resonant frequency ω_r, is read off at the tangent point on the $G(j\omega)$ curve.

N - circles

Consider $G(s)$ is the forward path transfer function of a unity feedback system.

The closed loop T/F is

$$M(s) = \frac{G(s)}{1+G(s)}.$$

For sinusoidal steady state, $s=j\omega$

$$G(j\omega) = Re(G(j\omega)) + j\,Im(G(j\omega))$$

$$= x + jy$$

where $x = Re[G(j\omega)]$, & $y = Im[G(j\omega)]$

Let α be the phase angle of closed loop system

$$M(j\omega) = \frac{x+jy}{1+x+jy}$$

$$\therefore \alpha = \tan^{-1}\left[\frac{y}{x}\right] - \tan^{-1}\left[\frac{y}{1+x}\right]$$

$$\Rightarrow \alpha = \tan^{-1}\left[\frac{\frac{y}{x} - \frac{y}{1+x}}{1+\frac{y}{x}\cdot\frac{y}{1+x}}\right]$$

$$\Rightarrow \tan\alpha = N\ (\text{say}) = \frac{\frac{y}{x} - \frac{y}{1+x}}{1+\frac{y}{x}\cdot\frac{y}{1+x}}$$

$$\Rightarrow N\left[1 + \frac{y^2}{x(1+x)}\right] = \frac{y(1+x) - yx}{x(1+x)}$$

$$\Rightarrow N\left[\frac{x(1+x)+y^2}{\cancel{x(1+x)}}\right] = \frac{y + \cancel{xy} - \cancel{xy}}{\cancel{x(1+x)}}$$

$$\Rightarrow N\left(x + x^2 + y^2\right] = y$$

$$\Rightarrow x^2 + x + y^2 = \frac{y}{N}$$

$$\Rightarrow x^2 + x + y^2 - \frac{y}{N} = 0$$

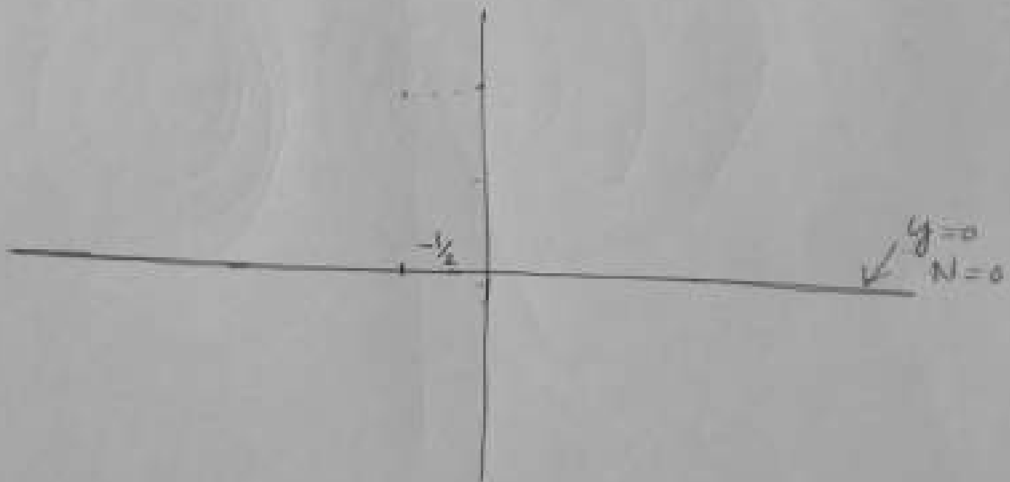
$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2N}\right)^2$$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(y - \cancel{\frac{1}{2N}}\right)^2 = \frac{1}{4} + \frac{1}{4N^2}$$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \cancel{\frac{N^2+4}{4N^2}} \left(\sqrt{\left(\frac{1}{4}\right) + \frac{1}{4N^2}}\right)^2$$

Centre of the circle: $\left(-\frac{1}{2}, +\frac{1}{2N}\right)$

Radius of the circle: $\sqrt{\left(\frac{1}{4}\right) + \frac{1}{4N^2}}$, $N \neq 0$



N	$\left(-\frac{1}{2}, \frac{1}{2N}\right)$	$\sqrt{\frac{1}{4} + \frac{1}{4N^2}}$	
$\neq$ 0	$\left(-\frac{1}{2}, \infty\right)$	∞	$\rightarrow y = 0 \; \forall x$
0.1	$\left(-\frac{1}{2}, 5\right)$	5.025	
0.5	$\left(-\frac{1}{2}, 1\right)$	1.12	
1	$\left(-\frac{1}{2}, 0.5\right)$	0.71	
2	$\left(-\frac{1}{2}, 0.25\right)$	0.612	

Previous Year Question Paper (1)

Control System [6th Sem Electrical]

1. Answer All questions (2 x 10)

(a) Define centroid and break away point in root locus.

Ans:- CENTROID :-

i) The intersection of the asymptotes lie only on the real axis of the s-plane.

ii) This point of intersection of the asymptotes is called centroid, given by

$$\sigma = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n-m}$$

where n is the number of finite poles of $G(s)H(s)$ & m is the number of finite zeros of $G(s)H(s)$.

BREAK AWAY POINT :- (i) Break-away point on the Root locus indicates ~~dum~~ the presence of multiple closed loop poles.

(ii) Break away point on the Root locus is determined by finding the roots of $\frac{dk}{ds} = 0$ or $\frac{d\,G(s)H(s)}{ds} = 0$.

(b) What is meant by resonant peak and Bandwidth of a system?

Ans:- RESONANT PEAK :- The resonant peak M_r is the maximum value of $|M(j\omega)|$

Where $|M(j\omega)| = \left| \dfrac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2} \right| = \dfrac{|G(j\omega)|}{|1+G(j\omega)|}$

M_r is given by,

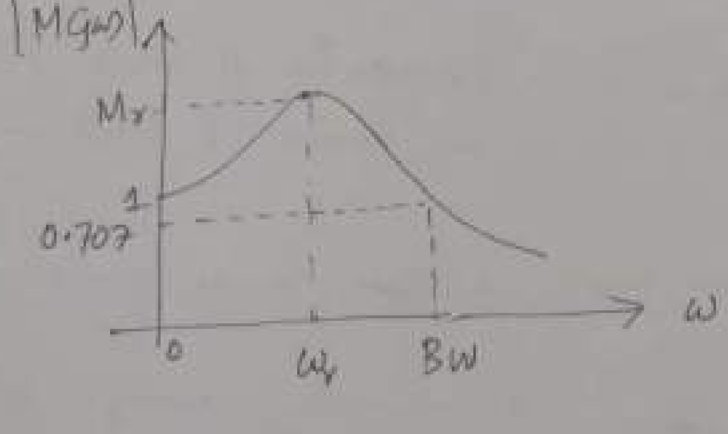
$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}, \quad \xi \le 0.707$$

Band Width :

Bandwidth is the frequency at which $|M(j\omega)|$ drops to 70.7% of, or 3 dB down from, its zero frequency value.

Bandwidth of 2nd Order proto type system is:

$$BW = \omega_n \sqrt{(1-2\xi^2) + \sqrt{(1-2\xi^2)^2 + 1}}$$



(c) State the formula to find out angle of arrival & angle of departure.

Ans: The angle of departure or arrival of RL from a pole or a zero of $G(s)H(s)$ can be determined by assuming a point s_1 that is very much close to the pole or zero & applying the equation,

$$\angle G(s_1)H(s_1) = \sum_{K=1}^{m} \angle(s_1+z_K) - \sum_{j=1}^{n} \angle(s_1+p_j)$$

$$= (2i+1)\times 180^\circ, \quad K \geq 0$$

$$= 2i \times 180^\circ, \quad K < 0$$

where $i = 0, \pm 1, \pm 2, \cdots$

d. What is Nichols chart ?

Ans: (I) Constant magnitude loci that are M-circles & constant Phase angle loci that are N circles are the fundamental components in designing Nichol's chart.

(II) The constant M and constant N circles in G(jω) plane can be used for the analysis & design of control system.

(III) Constant M and constant N circles are prepared in gain-phase plane. Gain phase plane is the graph having gain in decibel along ordinate and phase angle along abscissa.

(IV) The M & N circles of G(jω) in the gain phase plane are transformed into M & N contours in rectangular coordinates.

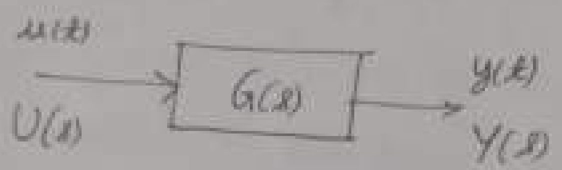
(V) A point on the constant M loci in G(jω) is transferred to a gain phase plane by drawing the vector directed from the origin of G(jω) plane to that point on M circles & then measuring the length in dB & angle in degree.

e. <u>Define Transfer function.</u>

<u>Ans</u>:→ For an LTI system, transfer function is the ratio of the Laplace transform of the output to the Laplace transform of the input with the initial conditions being zero.

→ Mathematically, if $U(s)$ is the Laplace transform of the input function & $Y(s)$ is the Laplace transform of the output the transfer function $G(s)$ is given by,

$$G(s) = \frac{Y(s)}{U(s)}$$



(f) What do you mean by polar plot ?

<u>Soln</u>:- Polar plot of $G(s)H(s)$ is a plot of $G(j\omega)H(j\omega)$ in the polar coordinates of $Im[G(j\omega)H(j\omega)]$ vs. $Re[G(j\omega)H(j\omega)]$ as ω varies from 0 to ∞.

(g) What is the effect of addition of zeros to Root Locus?

<u>Ans</u>:- (I) Adding left Half plane zeros to the function $G(s)H(s)$ generally has the effect of moving & bending the root loci towards left half s-plane.

(II) Relative stability of the system increases.

(III) System becomes less oscillatory

(IV) Gain margin increases so does the range of K.

(V) Settling time decreases.

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h. Define rise time and peak time of a system. ③

Soln:- RISE TIME :- Rise time for a 2nd Order unity feedback under damped system is the time required for the step response to increase from 0% to 100% of steady-state response for the first time.

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

Where $\cos\phi = \xi$, ξ is the damping ratio

$\omega_d = \omega_n \sqrt{1-\xi^2}$, ω_d is the damped frequency of oscillation.

$\omega_n \rightarrow$ undamped natural frequency of oscillation.

PEAK TIME :- Peak time for a 2nd Order unity feedback underdamped system is the time required for the step response to increase from zero to maximum value of response.

$$\boxed{t_p = \frac{\pi}{\omega_d}}$$

ⓘ What is Mason's Gain formula?

Ans: Given an SFG (Signal Flow Graph) with N forward paths and K loops, the Gain between input node y_{in} & output node y_{out} is

$$\frac{Y_{out}}{Y_{in}} = \frac{\sum_{K=1}^{N} P_K \Delta_K}{\Delta}$$

where,

Y_{in} = input-node variable

Y_{out} = output-node variable.

N = total number of forward paths between Y_{in} and Y_{out}.

P_K = Gain of the Kth Forward path between Y_{in} and Y_{out}.

Δ = 1 − (sum of the gains of all individual loops) + (sum of product of all possible combination of two non-touching loops) − (sum of product of all possible combination of three non-touching loops) + ...

Δ_K = the Δ for that part of the SFG that is non-touching with Kth Forward path.

(j) What is Routh's stability criteria?

(i) Routh's stability criterion is an algebraic method that provides information on the absolute stability of a linear-time invariant system that has a characteristics equation with constant coefficients

(ii) It determines the location of roots (zeros) of characteristics equation w.r.t. left half & right half of the s-plane.

Without actually solving for the roots (zeros). (4)

(iii) Consider that the characteristics equation of LTI SISO system is of the form.

$$F(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \quad -(1)$$

where all the coefficients are real.

(iv) In order that equation (1) doesnot have roots with positive real parts, it is necessary (but not sufficient) that the following conditions hold.

1. All the coefficients of equation (1) have same sign.
2. None of the coefficients vanishes

(v) Routh's tabulation

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

s^3	a_3	a_1
s^2	a_2	a_0
s^1	$\frac{a_2 a_1 - a_3 a_0}{a_2}$	0
s^0	a_0	0

The roots of the equation are all in the left half of the s-plane if all the elements of the first column of Routh's tabulation are of the same sign. The number of changes of sign in the elements of first column equals the number of roots with positive real parts or in the right half s-plane.

2. Answer any six questions.

(i) Determine the stability of a system whose characteristics equation

$$s^4+2s^3+10s^2+8s+4=0$$

Ans:- Given Characteristics equation,

$$s^4+2s^3+10s^2+8s+4=0 \quad \text{—(1)}$$

RH tabulation

s^4	1	10	4
s^3	2	8	0
s^2	6	4	0
s^1	$\frac{20}{3}$	0	0
s^0	4	0	0

Since equation (1) has no coefficient missing and all are real. All the elements of first column in RH table have same sign (+ve).

So the system is absolutely stable.

(1) Explain steady state error and error constants.

Soln:- <u>Steady State Error</u>

Error of a system is defined by,

$$e(t) = \text{reference signal} - y(t)$$

Where reference signal is the signal that the output $y(t)$ is to track.

When the system has unity feedback, i.e, $H(s)=1$ then the input $r(t)$ is the reference signal and

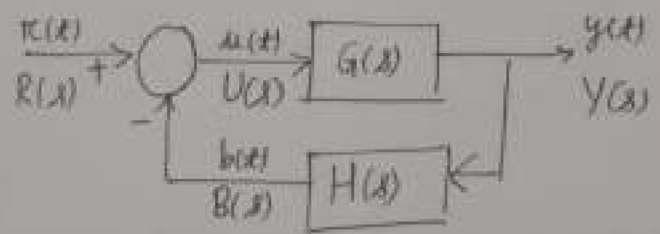
$$\boxed{e(t) = r(t) - y(t)}$$

The steady-state error is defined as:

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} \left[\text{reference signal} - y(t)\right]$$

$$= \lim_{t \to \infty} \left[r(t) - y(t)\right]$$



For an unity feedback system,

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}}$$

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Error Constants

(i) Position Error Constant or Step-error constant

$$\boxed{K_p = \lim_{s \to 0} G(s)}$$

(ii) Velocity Error Constant or Ramp-error constant

$$\boxed{K_v = \lim_{s \to 0} sG(s)}$$

(iii) Acceleration Error constant or Parabolic-error constant

$$\boxed{K_a = \lim_{s \to 0} s^2 G(s)}$$

Type of System	Error Constants			Steady-State Error e_{ss} Step input	Ramp input	Parabolic input
	K_p	K_v	K_a	$\frac{R}{1+K_p}$	$\frac{R}{K_v}$	$\frac{R}{K_a}$
0	K	0	0	$\frac{R}{1+K}$	∞	∞
1	∞	K	0	0	$\frac{R}{K}$	∞
2	∞	∞	K	0	0	$\frac{R}{K}$
3	∞	∞	∞	0	0	0

(iii) Derive rise time, maximum peak over shoot and settling time for second order under damped system.

Soln: The unit step response of second order unity negative feedback under-damped system is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[\omega_d t + \phi], \quad t \geq 0$$

Where $\cos\phi = \xi$ & $\omega_d = \omega_n\sqrt{1-\xi^2}$.

① RISE TIME (t_r)

It is the time required for response to increase from 0% to 100% of steady state value for the first time.

Steady state value of output is $\boxed{c(\infty) = 1}$

$$\therefore \quad 1 = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[\omega_d t + \phi]$$

$$\Rightarrow \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[\omega_d t + \phi] = 0$$

$$\Rightarrow \sin[\omega_d t + \phi] = 0 = \sin\pi$$

$$\Rightarrow \omega_d t + \phi = \pi$$

$$\Rightarrow t = \frac{\pi - \phi}{\omega_d}$$

$$\therefore \text{Rise time} \quad \boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

② MAXIMUM PEAK-OVERSHOOT

At maximum peak overshoot

$$\frac{dc(t)}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin[\omega_d t + \phi]\right] = 0$$

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$$\Rightarrow \left[0 - \frac{1}{\sqrt{1-\xi^2}}\left\{e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t+\varphi) + e^{-\xi\omega_n t}\cos(\omega_d t+\varphi)(\omega_d)\right\}\right] = 0$$

$$\Rightarrow \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\omega_d\cos(\omega_d t+\varphi) - \xi\omega_n\sin(\omega_d t+\varphi)\right] = 0$$

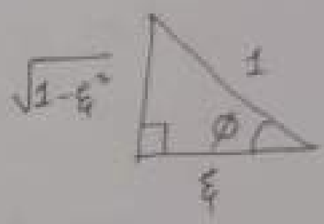
$$\Rightarrow \omega_d\cos(\omega_d t+\varphi) - \xi\omega_n\sin(\omega_d t+\varphi) = 0$$

$$\Rightarrow \frac{\sin(\omega_d t+\varphi)}{\cos(\omega_d t+\varphi)} = \frac{\omega_d}{\xi\omega_n} = \frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n}$$

$$\Rightarrow \tan[\omega_d t+\varphi] = \tan[\phi]$$

$$\Rightarrow \omega_d t+\varphi = n\pi+\varphi$$

$$\Rightarrow \omega_d t = n\pi$$



for maximum peak overshoot take $n=1$

$$\therefore \boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

And the peak overshoot is

$$M_p = c(t_p) - c(\infty)$$

$$= \left[1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}}\sin[\omega_d t_p+\varphi]\right] - 1$$

$$= \left[1 - \frac{e^{-\xi\omega_n \times \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}}\sin\left[\omega_d \times \frac{\pi}{\omega_d}+\varphi\right]\right] - 1$$

$$= \left[1 + \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \times \sin\varphi\right] - 1$$

$$= \left(1 + \frac{e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}}{\cancel{\sqrt{1-\xi^2}}} \times \cancel{\sqrt{1-\xi^2}}\right) - 1$$

$$\Rightarrow \boxed{M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}}$$

Settling time :- It is the time required for the response to remain within specified limits of steady state value for the first time.

Time period of oscillation is :-

$$T = \frac{1}{\xi\omega_n}$$

2% settling time is $4T = \frac{4}{\xi\omega_n}$

5% settling time is $T = \frac{1}{\xi\omega_n}$

1) Discuss the correlation between time domain & frequency domain specifications.

Soln:-

$$r(t) \longrightarrow \boxed{\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}} \longrightarrow y(t)$$

Prototype Second Order System.

Time-Domain Specification	Frequency-Domain Specification
→ $y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \varphi)$, $t \geq 0$ $\omega_d = \omega_n\sqrt{1-\xi^2}$ $\cos\varphi = \xi$	→ $M(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$
→ Peak Overshoot, $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$ $0 < \xi < 1$	→ Resonant Peak, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$, $\xi \leq 0.707$

Time-Domain Specification

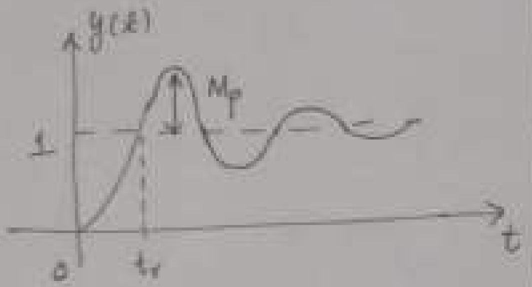
→ Damped frequency of oscillation,

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

→ Rise time,

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_n\sqrt{1-\xi^2}}$$

→



→ As ω_n gets larger, t_r gets smaller and the system responds faster.

→ As ξ gets larger, t_r gets larger and the system responds slower.

Frequency-Domain Specification

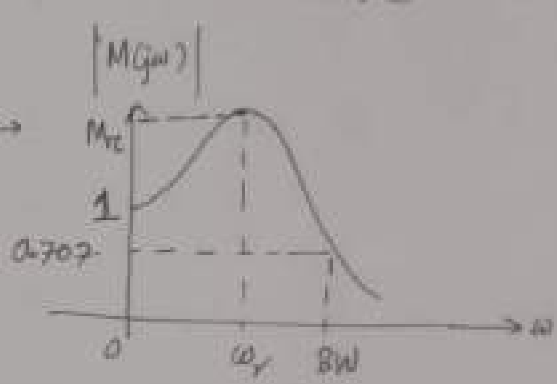
→ Resonant Frequency,

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

→ Bandwidth,

$$BW = \omega_n\sqrt{(1-2\xi^2) + \sqrt{(1-2\xi^2)^2+1}}$$

→



→ As ω_n gets larger, BW gets larger

→ As ξ gets larger, BW gets smaller.

Conclusion :

→ BW and rise time are inversely proportional.

→ Increasing ω_n, increases BW & decreases t_r

→ Increasing ξ, decreases BW & increases t_r

V. Draw the polar plot of given open loop system

$$G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$

Soln:- Given $G(s) = \frac{K}{(1+sT_1)(1+sT_2)} = K\, G_1(s)$

Put $s = j\omega$

$$G_1(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$\therefore \quad |G_1(j\omega)| = \frac{1}{\sqrt{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}}$$

$$\angle G_1(j\omega) = \left(-\tan^{-1}(\omega T_1)\right) + \left(-\tan^{-1}(\omega T_2)\right)$$

$$\therefore \quad G_1(j\omega) = \frac{(1-j\omega T_1)(1-j\omega T_2)}{(1+j\omega T_1)(1-j\omega T_1)(1+j\omega T_2)(1-j\omega T_2)}$$

$$= \frac{(1-\omega^2 T_1 T_2) - j\omega(T_1+T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$\therefore \quad Re\left[G_1(j\omega)\right] = \frac{(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

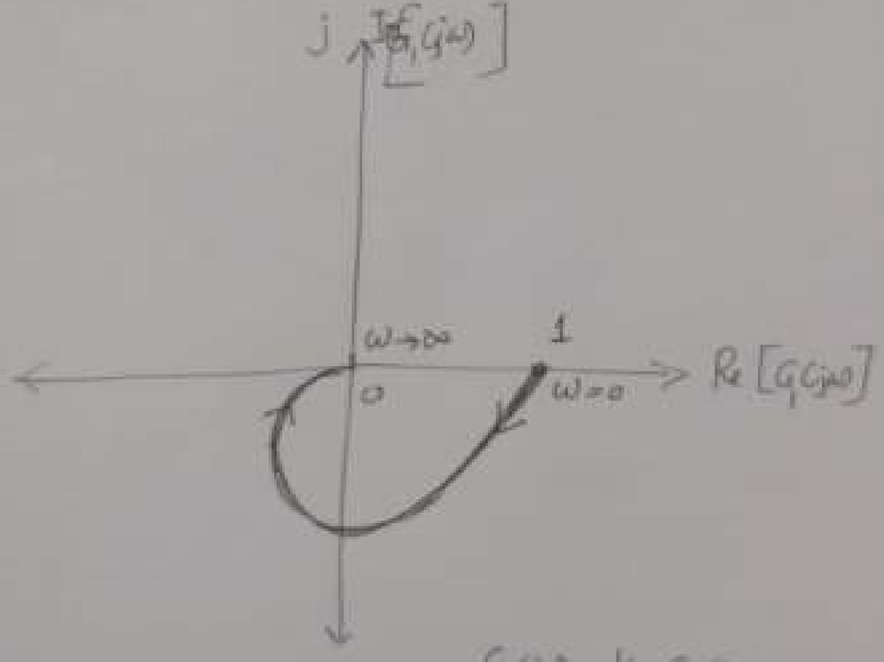
$$Im\left[G_1(j\omega)\right] = -\frac{\omega(T_1+T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

On Real axis $Im\left[G_1(j\omega)\right] = 0$

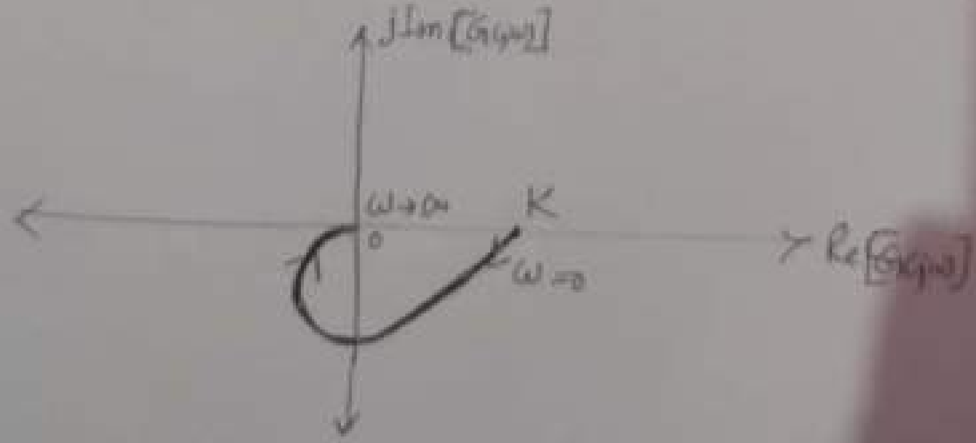
$$\Rightarrow \quad \omega = 0$$

$$\therefore \quad Re\left[G_1(j\omega)\right]\Big|_{\omega=0} = \frac{1-0}{(1+0)(1+0)} = 1$$

ω	$\lvert G_1(j\omega)\rvert = \frac{1}{\sqrt{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}}$	$\angle G_1(j\omega) = -(\tan^{-1}\omega T_1 + \tan^{-1}\omega T_2)$	$Re[G_1(j\omega)] = \frac{1-\omega^2T_1T_2}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$	$Im[G_1(j\omega)] = -\frac{\omega(T_1+T_2)}{(1+\omega^2T_1^2)(1+\omega^2T_2^2)}$
0	1	0°	1	-0
∞	0	-180°	$\lim_{\omega\to\infty}\frac{\omega^2\left[\frac{1}{\omega^2}-T_1T_2\right]}{\omega^4\left(\frac{1}{\omega^2}+T_1^2\right)\left(\frac{1}{\omega^2}+T_2^2\right)}$ $= \frac{(0-T_1T_2)}{\infty(0+T_1^2)(0+T_2^2)}$ $= -\frac{T_1T_2}{\infty}$ $= -0$	-0



$G(j\omega) = K\,G_1(j\omega)$



(vi) Find error-coefficients and steady state error of a unity feedback system whose open loop transfer function is given by $G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$, when subjected to an input given by $r(t) = 2 + 5t + 2t^2$

Soln: Error coefficients :-

a) Position error constant,

$$K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$= \frac{108}{0^2(0+4)(0^2+3\times0+12)}$$

$$= \frac{108}{0} = \infty$$

$$\therefore \boxed{K_p = \infty}$$

(b) Velocity Error Constant

$$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} \frac{\cancel{s} \times 108}{s^2(s+4)(s^2+3s+12)}$$

$$= \lim_{s \to 0} \frac{108}{s(s+4)(s^2+3s+12)}$$

$$= \frac{108}{0 \times (0+4) \times (0^2+3\times0+12)}$$

$$= \frac{108}{0} = \infty$$

$$\therefore \boxed{K_v = \infty}$$

(c) Acceleration error constant (K_a)

$$\therefore K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \frac{s^2 \times 108}{s^2(s+4)(s^2+3s+12)}$$

$$= \lim_{s\to 0} \frac{108}{(s+4)(s^2+3s+12)}$$

$$= \frac{108}{(0+4)(0^2+3\times 0+12)} = \frac{\cancel{108}^{\,9}}{4\times\cancel{12}} = \frac{9}{4}.$$

$$\therefore \boxed{K_a = \frac{9}{4}}$$

Steady-state error

Given input $r(t) = 2+5t+2t^2$

$$= 2u(t) + 5tu(t) + 4\tfrac{1}{2}t^2 u(t)$$

$$\therefore R(s) = \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3} = \frac{2s^2+5s+4}{s^3}$$

$\therefore$ steady-state error,

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{\cancel{s}\times\left(\frac{2s^2+5s+4}{s^{\cancel{3}\,2}}\right)}{1+\frac{108}{s^2(s+4)(s^2+3s+12)}}$$

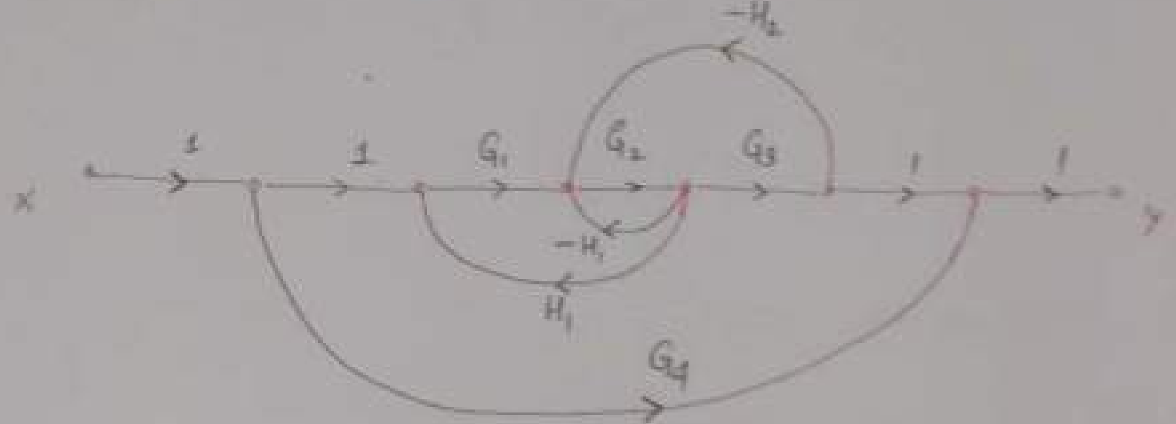
$$= \lim_{s\to 0} \frac{2s^2+5s+4}{s^2\left[1+\frac{108}{s^2(s+4)(s^2+3s+12)}\right]}$$

$$= \lim_{s\to 0} \frac{2s^2+5s+4}{s^2+\frac{108}{(s+4)(s^2+3s+12)}} = \frac{4}{\frac{108}{4\times 12}}$$

$$\Rightarrow e_{ss} = \frac{4 \times 4 \times 12}{\frac{108}{9}} = \frac{16}{9}$$

$$\therefore \boxed{e_{ss} = \frac{16}{9}}$$

(VI) Find the value of $\frac{Y}{X}$ of the SFG given in Fig. below:-



Soln:-

Forward Path

$P_1 = 1 \times 1 \times G_1 \times G_2 \times G_3 \times 1 \times 1 = G_1 G_2 G_3$

$P_2 = 1 \times G_4 \times 1 = G_4$

Single loop

$L_1 = G_1 G_2 H_1$

$L_2 = -G_2 H_1$

$L_3 = -G_2 G_3 H_2$

2 non-touching loops

doesnot exist.

$\Delta = 1 - (L_1 + L_2 + L_3)$

$= 1 - (G_1 G_2 H_1 - G_2 H_1 - G_2 G_3 H_2)$

$\Rightarrow \Delta = 1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2$

$\Delta_1 = 1 - (0) = 1$

$\Delta_2 = 1 - (L_1 + L_2 + L_3)$

$\Rightarrow \Delta_2 = 1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2$

By Mason's gain formula,

$$\frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{(G_1 G_2 G_3) \times 1 + G_4 [1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2]}{1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2}$$

$$= \frac{G_1 G_2 G_3 + G_4 - G_1 G_2 G_4 H_1 + G_2 G_4 H_1 + G_2 G_3 G_4 H_2}{1 - G_1 G_2 H_1 + G_2 H_1 + G_2 G_3 H_2}$$

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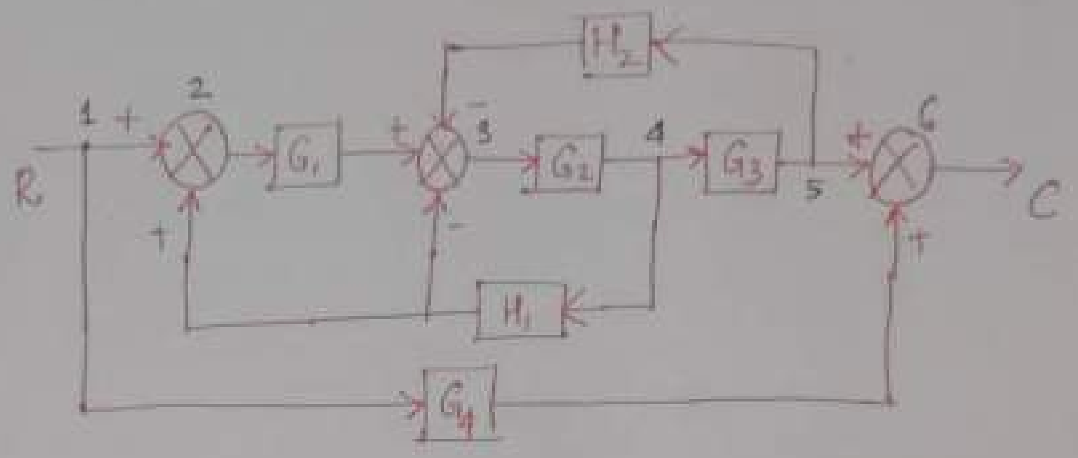
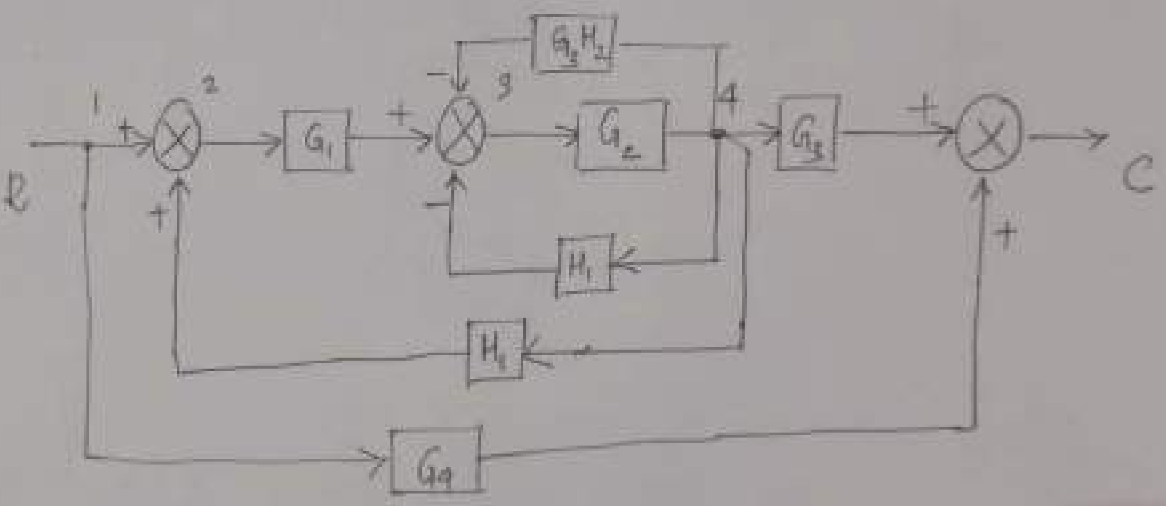
(5) Using Block diagram reduction technique, find $\frac{C(s)}{R(s)}$ of the system given in fig.

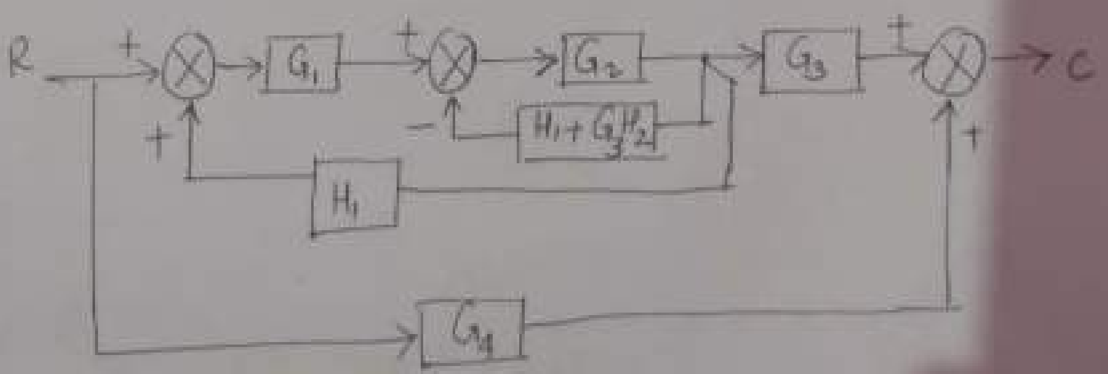


Soln:-

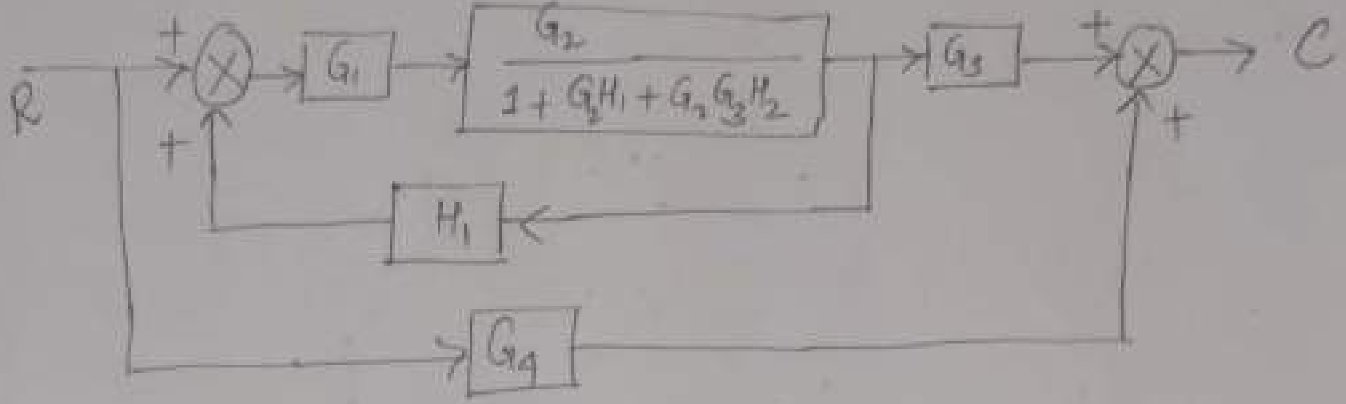
Step ①



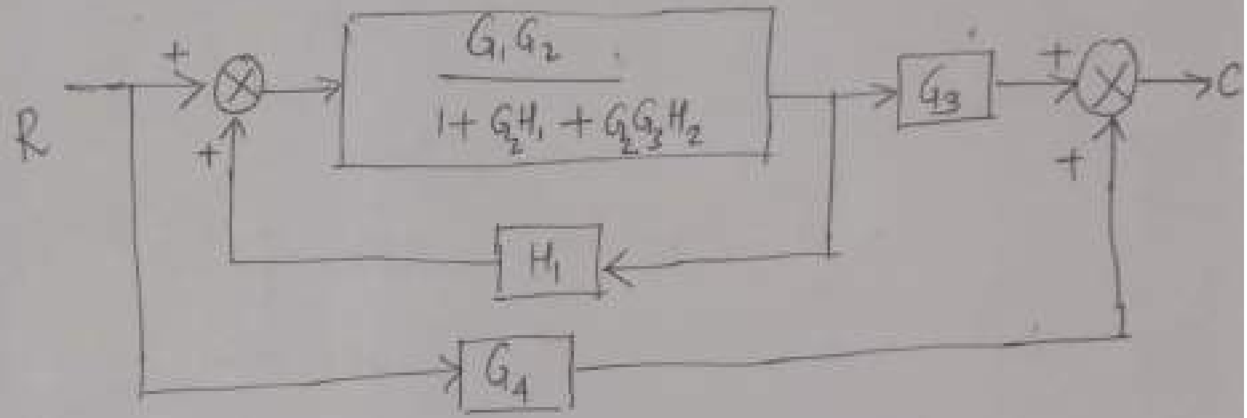
Step ②



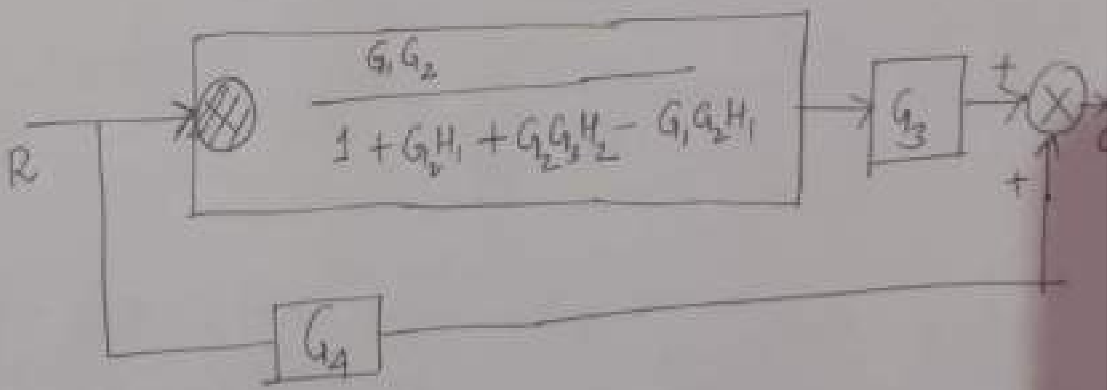
Step ③



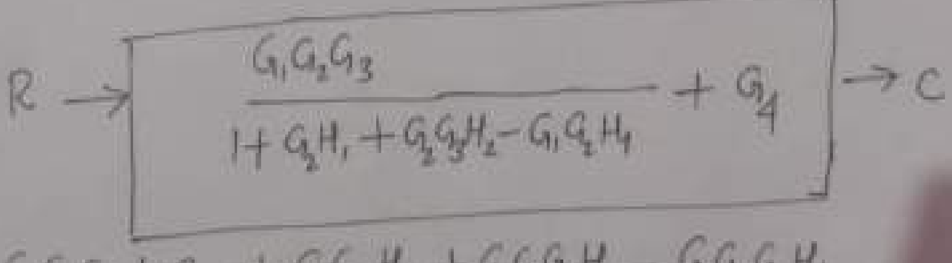
Step ④



Step ⑤ :



Step ⑥ :



$$\therefore \frac{C}{R} = \frac{G_1G_2G_3 + G_4 + G_2G_4H_1 + G_2G_3G_4H_2 - G_1G_2G_4H_1}{1 + G_2H_1 + G_2G_3H_2 - G_1G_2H_1}$$

(4) Sketch the Root locus of $G(s)H(s) = \frac{K}{s(s+5)(s+10)}$

Also mention about its stability.

Soln:- Given $G(s)H(s) = \frac{K}{s(s+5)(s+10)}$.

(i) Finite poles :- 0, −5, −10

No of finite poles, $\boxed{P=3}$

Finite Zeros : No zero

No of finite zeros ; $\boxed{Z=0}$

(ii) No of asymptotes $= |P-Z| = |3-0| = 3$

(iii) Characteristics equation

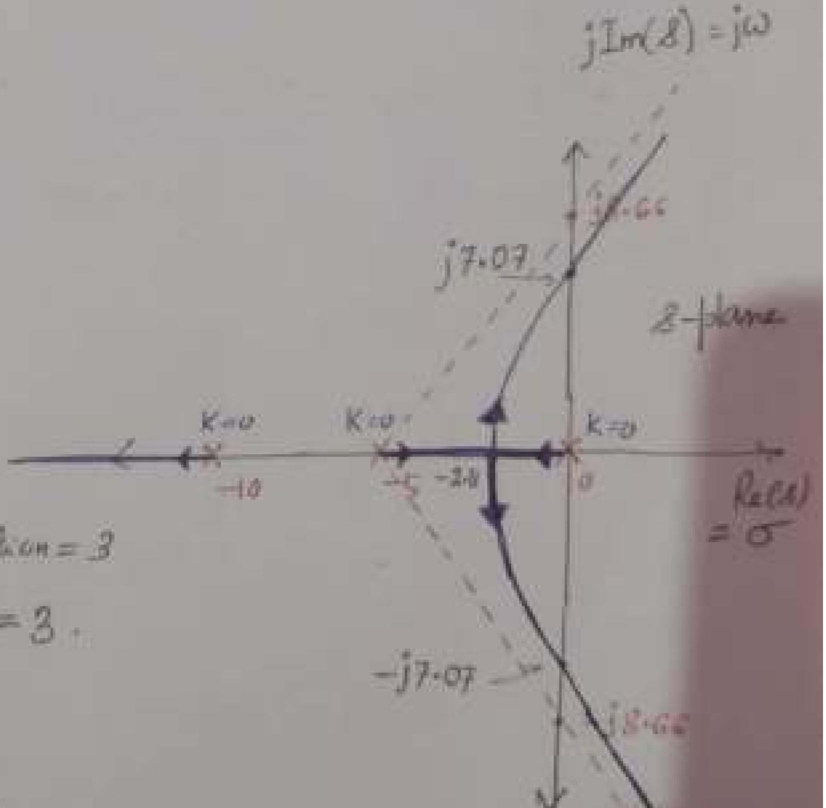
$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+5)(s+10)} = 0$$

$$\Rightarrow s(s+5)(s+10) + K = 0$$

$$\Rightarrow s\left[s^2 + 15s + 50\right] + K = 0$$

$$\Rightarrow s^3 + 15s^2 + 50s + K = 0$$



Order of characteristics equation = 3

⇒ No of root locus branches = 3.

(iv) Centroid

$$\sigma = \frac{0 + (-5) + (-10) - 0}{3 - 0}$$

$$= \frac{-15}{3} = -5$$

$0 \le K < \infty$

(v) Angle of asymptotes.

$$= \frac{(2i+1)\times 180^\circ}{|P-Z|}, \quad i = 0, 1, 2$$

$\therefore$ $i=0 \longrightarrow \frac{(2\times 0+1)180^\circ}{3} = 60^\circ$

$i=1 \longrightarrow \frac{(2\times 1+1)\times 180^\circ}{3} = 180^\circ$

$i=2 \longrightarrow \frac{(2\times 2+1)\times 180^\circ}{3} = 300^\circ$

(VI) Break - away point

$$\frac{dk}{ds} = 0 \Rightarrow \frac{d}{ds}\left[-(s^3+15s^2+50s)\right] = 0$$

$$\Rightarrow 3s^2 + 30s + 50 = 0$$

$$\Rightarrow s = -2.11 \ \& \ -7.89$$

Considering -2.11 as it lie on the root locus branch.

(VII) Intersection with imaginary axis

RH - table:-

s^3	1	50
s^2	15	k
s^1	$\frac{15\times 50 - k}{15}$	0
s^0	k	0

$\therefore \frac{15\times 50 - k}{15} = 0$

$\Rightarrow k = 15\times 50 = 750$

Auxiliary equation:-

$A(s) = 15s^2 + 750 = 0$

$\Rightarrow s^2 = -\frac{750}{15} = -50$

$\Rightarrow s = \sqrt{-50} = \pm j\,7.07$

(VIII) Angle of departure at break - away point

$$\pm\frac{180^\circ}{n} = \pm\frac{180^\circ}{2} = \pm 90^\circ$$

(IX) The System is absolutely stable for $\boxed{0 < K < 750}$
Marginally stable for $\boxed{K = 750}$ & unstable for $K > 750$

(5) Apply Nyquist stability criteria to Qn. (4) & mention about its stability.

Soln:- Given $G(s)H(s) = \frac{K}{s(s+5)(s+10)}$;

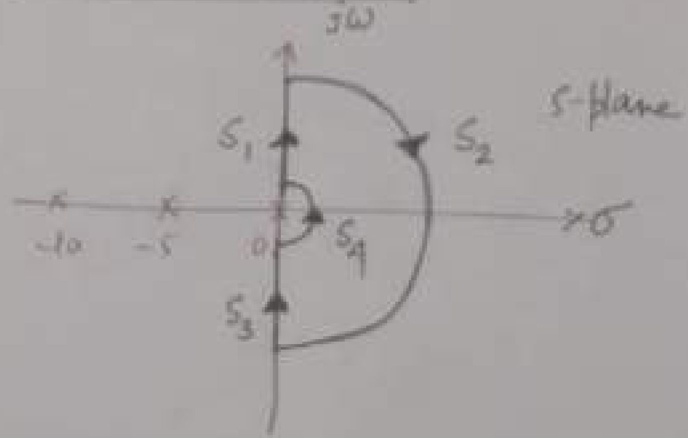
Type - 1, Order → 3

Open loop finite poles are at 0, -5, -10.

⇒ No of open loop poles to the Right Half s-plane,

$$\boxed{P = 0}$$

Nyquist Contour



Region S_1 :- ω varies from 0+ to ∞
It gives the polar plot

POLAR PLOT :- $G(s)H(s) = \frac{K}{s(s+5)(s+10)}$

$$= \frac{K}{50\,s(1+\frac{s}{5})(1+\frac{s}{10})}$$

$$= \frac{K/50}{s(1+\frac{s}{5})(1+\frac{s}{10})}$$

$$= (K/50)\, G_1(s)H_1(s)$$

$$G_1(s)H_1(s) = \frac{1}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)}$$

Put $s = j\omega$

$$G_1(j\omega)H_1(j\omega) = \frac{1}{j\omega\left(1+j\frac{\omega}{5}\right)\left(1+j\frac{\omega}{10}\right)}$$

$$\left|G_1(j\omega)H_1(j\omega)\right| = \frac{1}{\omega\sqrt{\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}}$$

$$\angle G_1(j\omega)H_1(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$G_1(j\omega)H_1(j\omega) = \frac{-j\left(1-j\frac{\omega}{5}\right)\left(1-j\frac{\omega}{10}\right)}{\omega\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}$$

$$= \frac{-j\left[1-\frac{\omega^2}{50}-j\frac{3\omega}{10}\right]}{\omega\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}$$

$$= \frac{-\frac{3\omega}{10}}{\omega\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)} - \frac{j\left(1-\frac{\omega^2}{50}\right)}{\omega\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}$$

$$= \frac{-0.3}{\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)} - \frac{j\left(1-\frac{\omega^2}{50}\right)}{\omega\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}$$

$$\mathrm{Re}\left[G_1(j\omega)H_1(j\omega)\right] = \frac{-0.3}{\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}, \quad \mathrm{Im}\left[G_1(j\omega)H_1(j\omega)\right] = \frac{-\left(1-\omega^2/50\right)}{\omega\left(1+\frac{\omega^2}{25}\right)\left(1+\frac{\omega^2}{100}\right)}$$

ω	$\lvert G_1(j\omega)H_1(j\omega)\rvert = \frac{1}{\omega\sqrt{(1+\frac{\omega^2}{25})(1+\frac{\omega^2}{100})}}$	$\angle G_1(j\omega)H_1(j\omega) = -90^\circ - \tan^{-1}(\frac{\omega}{5}) - \tan^{-1}(\frac{\omega}{10})$	$Re[G_1(j\omega)H_1(j\omega)] = \frac{-0.3}{(1+\frac{\omega^2}{25})(1+\frac{\omega^2}{100})}$	$Im[G_1(j\omega)H_1(j\omega)] = \frac{-(1-\omega^2/50)}{\omega(1+\frac{\omega^2}{25})(1+\frac{\omega^2}{100})}$
0	∞	-90°	-0.3	$-\infty$
∞	0	-270°	-0	$\lim_{\omega\to\infty} \frac{-\omega^2(\frac{1}{\omega^2}-\frac{1}{50})}{\omega^5(\frac{1}{\omega^2}+\frac{1}{25})(\frac{1}{\omega^2}+\frac{1}{100})}$ $= +0$

Intersection with Real axis

$$Im[G_1(j\omega)H_1(j\omega)] = 0$$

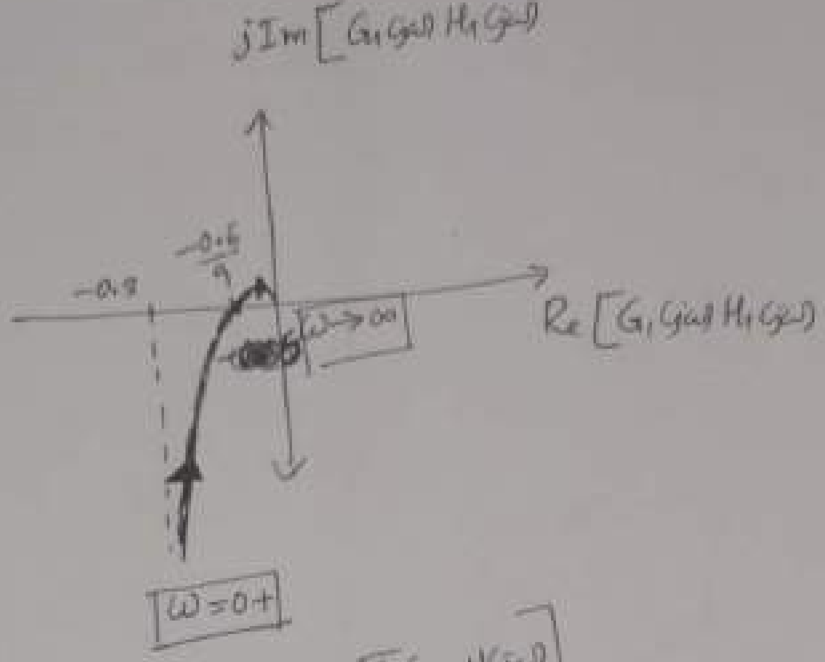
$$\Rightarrow 1 - \frac{\omega^2}{50} = 0$$

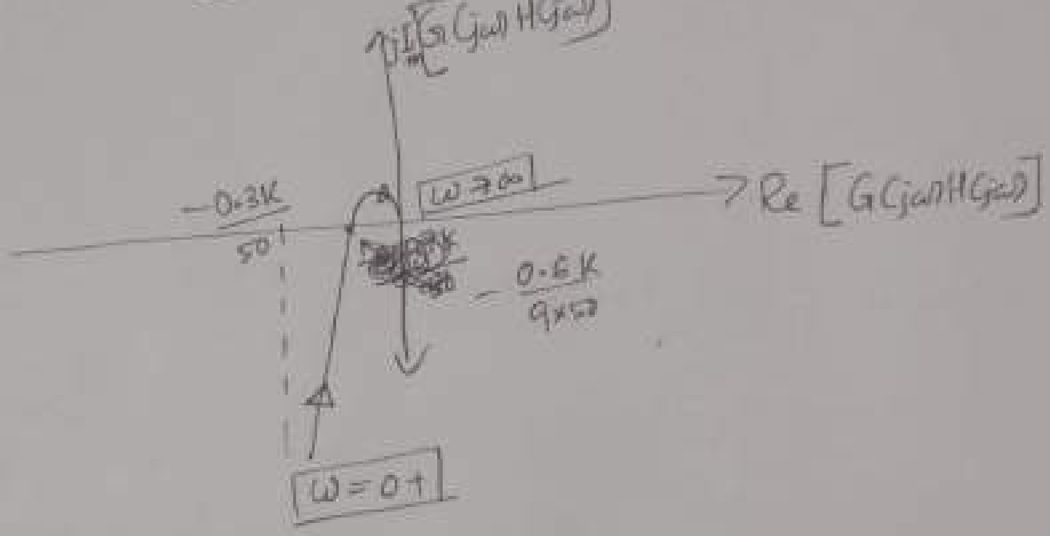
$$\Rightarrow \omega = \sqrt{50}$$

$$Re[G_1(j\omega)H_1(j\omega)]\Big|_{\omega=\sqrt{50}} = \frac{-0.3}{(1+\frac{50}{25})(1+\frac{50}{100})}$$

$$= \frac{-0.3}{3\times\frac{3}{2}}$$

$$= \frac{-0.3\times 2}{9} = -\frac{0.6}{9} = \cancel{-0.07}$$





Region S_3 : ω varies from $-\infty$ to $0-$

It gives the mirror image of polar plot about real axis.

Region S_4 : $s = \lim_{R\to\infty} Re^{j\theta}$

$\theta \to +\frac{\pi}{2}$ to $-\frac{\pi}{2}$

$$\frac{1}{s(1+\frac{s}{5})(1+\frac{s}{10})} \simeq \frac{1}{s^3} = \frac{1}{\lim\limits_{\substack{R\to\infty \\ \theta\to+\frac{\pi}{2}\text{ to }-\frac{\pi}{2}}} (Re^{j\theta})^3} = \frac{1}{(\infty)^3} e^{-j\left[\frac{3\pi}{2}\text{ to }-\frac{3\pi}{2}\right]}$$

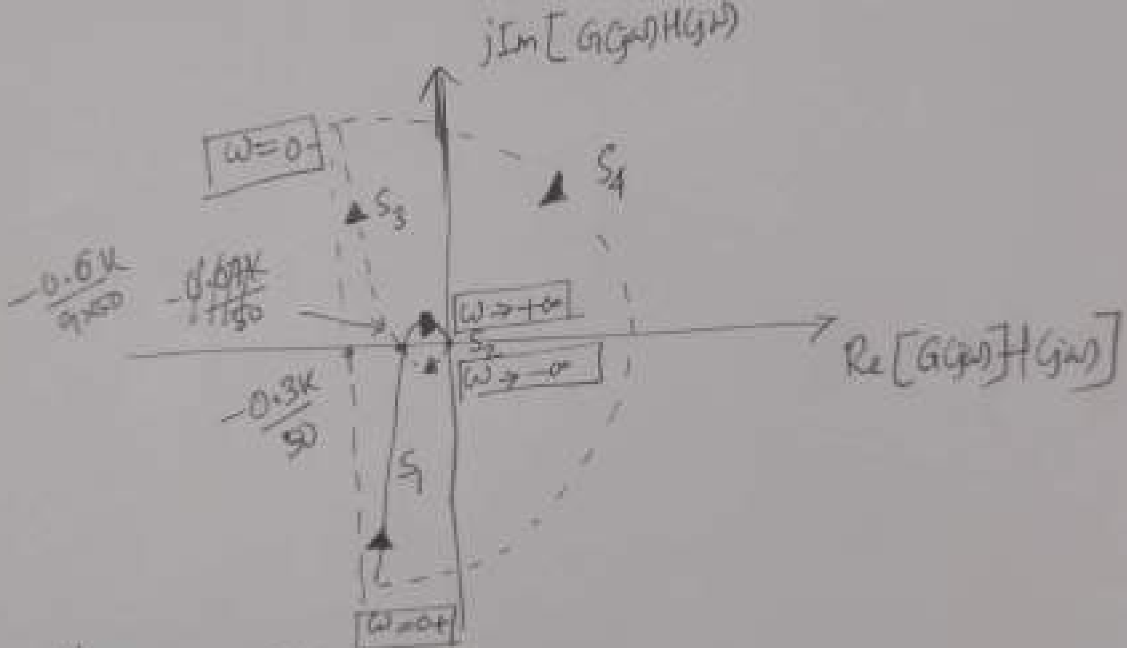
$$= 0\, e^{j\left[-\frac{3\pi}{2} \text{ to } +\frac{3\pi}{2}\right]}$$

Region S_4 :-

$$s = \lim_{\substack{R \to 0 \\ \theta \to -\frac{\pi}{2} \text{ to } \frac{\pi}{2}}} Re^{j\theta}$$

$$\frac{1}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)} \simeq \frac{1}{s} = \frac{1}{\lim\limits_{\substack{R \to 0 \\ \theta \to -\frac{\pi}{2} \text{ to } \frac{\pi}{2}}}} = \frac{1}{0} e^{-j\left(-\frac{\pi}{2} \text{ to } \frac{\pi}{2}\right)}$$

$$= \infty\, e^{j\left(\frac{\pi}{2} \text{ to } -\frac{\pi}{2}\right)}$$



Nyquist Stability Criterion

Given $\boxed{P = 0}$

For system to b stable

no of closed loop poles to the Right half s-plane should be = 0.

$\Rightarrow \boxed{Z = 0}$

$\therefore\ N = P - Z$

$= 0 - 0 \Rightarrow \boxed{N = 0}$

No of encirclements of critical point should be 0.

So. $\frac{0.6K}{50 \times 9} < 1$

$\Rightarrow 0.6K < 50 \times 9$

$\Rightarrow K < \frac{50 \times 9}{0.6}$

$\Rightarrow \boxed{K < 750}$

$\therefore 0 < K < 750$, for system to be absolutely stable

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(6) A unity feedback control system has

$$G(s) = \frac{80}{s(s+2)(s+20)}$$

Draw the Bode plot.

Determine GM, PM, ω_{gc}, ω_{pc}. Also determine its stability.

Soln:-

Given $G(s) = \frac{80}{s(s+2)(s+20)}$

$$= \frac{80}{s \times 2(1+\frac{s}{2}) \times 20 \times (1+\frac{s}{20})}$$

$$= \frac{\frac{80}{2 \times 20}}{s(1+\frac{s}{2})(1+\frac{s}{20})}$$

$$= \frac{2}{s(1+\frac{s}{2})(1+\frac{s}{20})} \quad \text{(Time constant form)}$$

$$\Rightarrow G(j\omega) = \frac{2}{j\omega(1+j\frac{\omega}{2})(1+j\frac{\omega}{20})}$$

$K = 2 \longrightarrow 20\log_{10} K = 20\log_{10} 2 = 6.02$ dB.

$\frac{1}{s} \longrightarrow$ initial slope $= -20$ dB/decade

$\frac{1}{(1+\frac{s}{2})^1} \rightarrow$ corner frequency $= 2$ rad/sec $= \omega_1 \longrightarrow$ change of slope at $\omega_1 = 2$ rad/sec is -20 dB/dec.

$\frac{1}{(1+\frac{s}{20})^1} \rightarrow$ Corner frequency $\omega_2 = 20$ rad/sec $\rightarrow$ change of slope at $\omega_2 = 20$ rad/sec $= -20$ dB/dec.

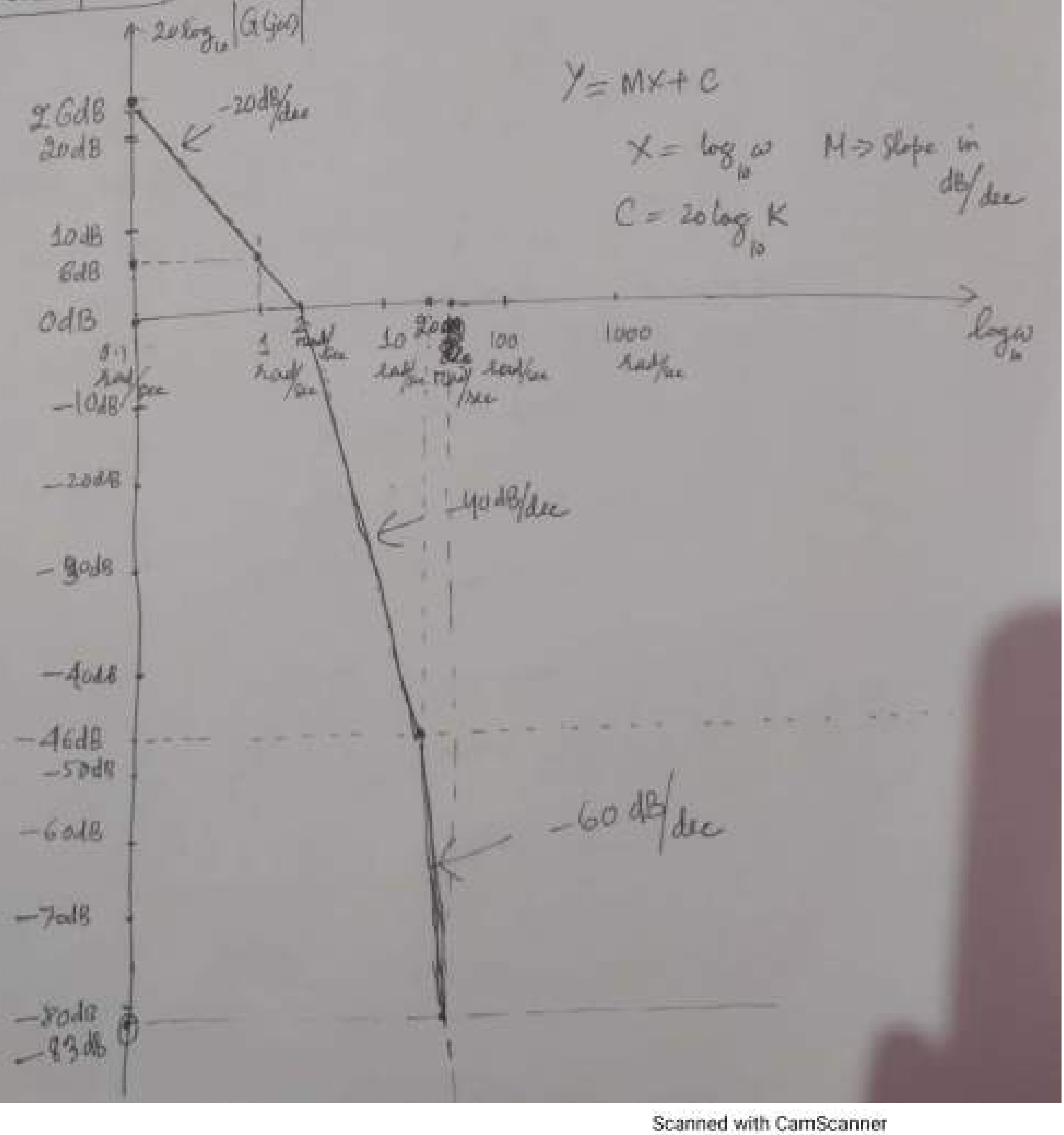
$$\angle G(j\omega) = -90° - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$$

Bode

ω		$-90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$
0.1	$\longrightarrow$	$-90^\circ - 2.86 - 0.29^\circ = -93.15^\circ$
0.5	$\longrightarrow$	$-90^\circ - 14.04^\circ - 1.43^\circ = -105.47$
1	$\longrightarrow$	$-90^\circ - 26.56^\circ - 2.86 = -119.42$
2	$\longrightarrow$	$-90^\circ - 45^\circ - 5.71^\circ = -140.71^\circ$
5	$\longrightarrow$	$-90^\circ - 68.20^\circ - 14.04^\circ = -172.24^\circ$
10	$\longrightarrow$	$-90 - 78.69^\circ - 26.56 = -195.25^\circ$
100	$\longrightarrow$	$-90^\circ - 88.85^\circ - 78.69^\circ = -257.54^\circ$
$\vdots$		
∞	$\longrightarrow$	$-90^\circ - 90^\circ - 90^\circ = -270^\circ$

Bode plot:

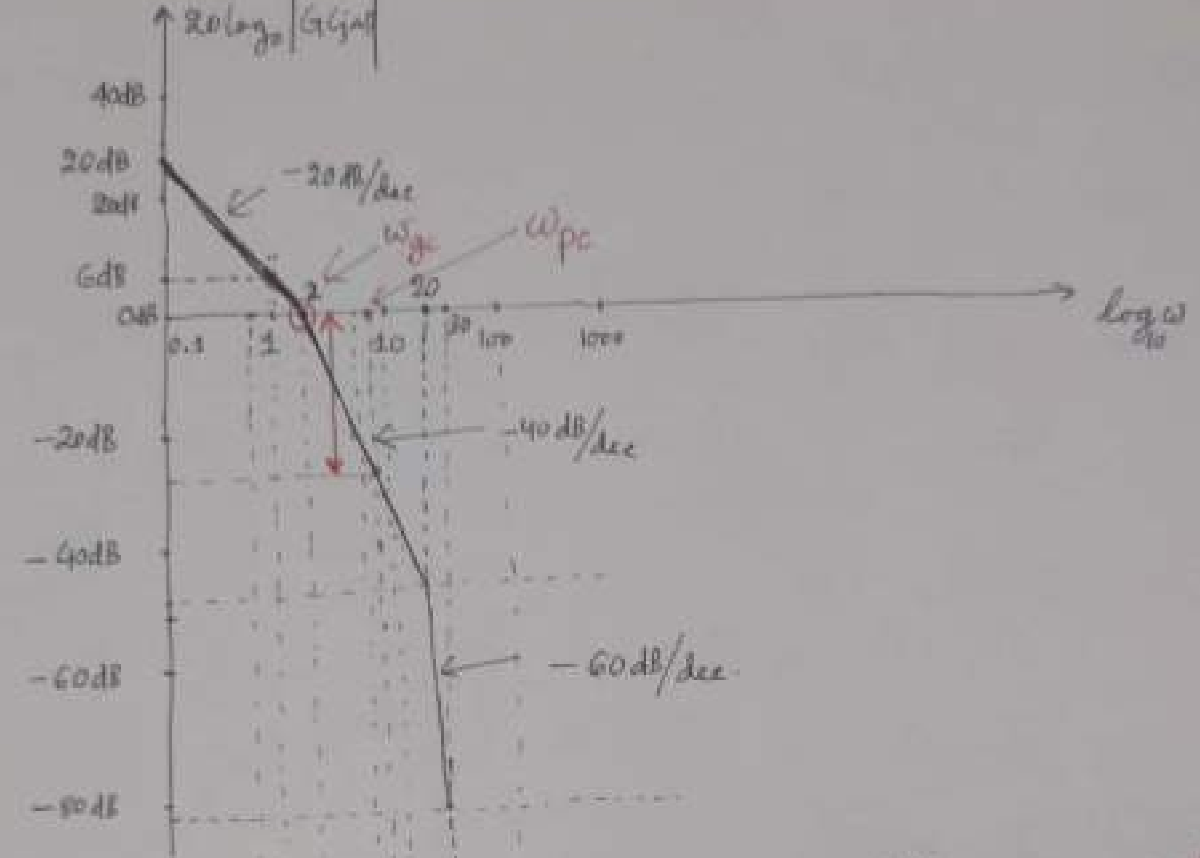
MAGNITUDE PLOT

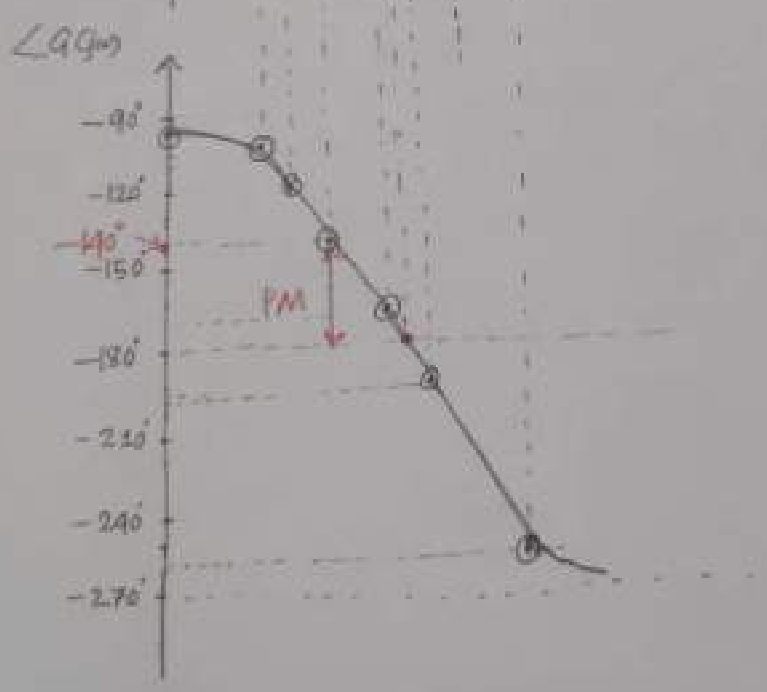


$Y = MX + C$

$X = \log_{10}\omega$ $\quad M \Rightarrow$ Slope in dB/dec

$C = 20\log_{10} K$





When you solve these problems using semi-log graph paper you will get values much closer to mathematically calculated values.

Even with these values you can comment on stability of a system.

From the graph $\omega_{gc} \simeq 2$ rad/sec [Frequency at which 0dB line cuts the Bode magnitude plot]

$-140° - (-180°) = PM \simeq 40°$

$\omega_{pc} \simeq 6$ rad/sec [Frequency at which -180° line cuts the Bode phase plot]

$0\,dB - (-30\,dB) = GM \simeq 30\,dB$

$\omega_{pc} > \omega_{gc}$ & both GM & PM are positive $\Rightarrow$ System is absolutely stable.

Mathematical evaluation of GM, PM, ω_{pc} & ω_{gc}

Soln:- Given $G(s) = \dfrac{80}{s(s+2)(s+20)}$

Put $s = j\omega$

$$G(j\omega) = \frac{80}{j\omega\,(j\omega+2)(j\omega+20)}$$

$$|G(j\omega)| = \frac{80}{\omega\sqrt{(\omega^2+4)(\omega^2+400)}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right)$$

Gain Margin (GM)

Phase cross-over frequency (ω_{pc})

$$\angle G(j\omega) = -180^\circ$$

$$\Rightarrow -90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{20}\right) = -180^\circ$$

$$\Rightarrow \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{20}\right) = 90^\circ$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{\omega}{2}+\frac{\omega}{20}}{1-\frac{\omega}{2}\times\frac{\omega}{20}}\right] = 90^\circ$$

$$\Rightarrow \frac{\frac{\omega}{2}+\frac{\omega}{20}}{1-\frac{\omega^2}{40}} = \tan 90^\circ = \infty$$

$$\Rightarrow 1-\frac{\omega^2}{40} = 0 \Rightarrow \boxed{\omega_{pc} = \sqrt{40}} = 6.32 \text{ rad/sec}$$

$$x = |G(j\omega)|\Big|_{\omega=\omega_{pc}=\sqrt{40}} = \frac{80}{\sqrt{40}\sqrt{(4+40)(400+40)}}$$

$$= \frac{80}{\sqrt{40\times 44\times 440}} = \frac{80}{880} = \frac{1}{11}$$

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$\therefore$ GM $= \frac{1}{x} = \frac{1}{1/11} = 11$

GM(dB) $= 20 \log_{10} 11 = 20.83$ dB

Phase Margin (PM)

Gain Cross-Over Frequency (ω_{gc})

$$|G(j\omega)| = 1$$

$$\Rightarrow \frac{80}{\omega\sqrt{(\omega^2+4)(\omega^2+400)}} = 1$$

$$\Rightarrow 80 = \omega\sqrt{(\omega^2+4)(\omega^2+400)}$$

$$\Rightarrow 6400 = \omega^2(\omega^2+4)(\omega^2+400)$$

$$\Rightarrow \omega^2\left[\omega^4 + 404\omega^2 + 1600\right] - 6400 = 0$$

$$\Rightarrow \omega^6 + 404\omega^4 + 1600\omega^2 - 6400 = 0$$

Say $\omega^2 = x$

$$\therefore x^3 + 404x^2 + 1600x - 6400 = 0$$

$$\Rightarrow x = 2.46$$

$$\Rightarrow \omega^2 = 2.46$$

$$\Rightarrow \omega = 1.57 \Rightarrow \boxed{\omega_{gc} = 1.57 \text{ rad/sec}}$$

$$\therefore \phi\Big|_{\omega=\omega_{gc}=1.57 \text{ rad/sec}} = -90^\circ - \tan^{-1}\left(\frac{1.57}{2}\right) - \tan^{-1}\left(\frac{1.57}{20}\right)$$

$$= -90^\circ - 38.13^\circ - 4.49^\circ$$

$$= -132.62^\circ$$

$$PM = 180^\circ - |\phi| = 180^\circ - |-132.62^\circ| = 47.38^\circ$$

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(7) Write short notes on

a) Constant M circles & Constant N circles.
b) All pass & Minimum Phase System
c) PID controller

Soln:-

(a) Constant M circles and Constant N circles

Constant M circles :- Consider $G(s)$ is the forward path transfer function of a unity feedback system. The closed loop T/F is:

$$M(s) = \frac{G(s)}{1+G(s)}$$

For sinusoidal steady state, $s = j\omega$

$$G(j\omega) = Re[G(j\omega)] + j\,Im[G(j\omega)]$$
$$= x + jy$$

where $x = Re[G(j\omega)]$ & $y = Im[G(j\omega)]$

Magnitude of closed loop t/f is:-

$$|M(j\omega)| = \left|\frac{G(j\omega)}{1+G(j\omega)}\right| = \left|\frac{x+jy}{1+x+jy}\right| = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$$

$$\Rightarrow M = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}} \Rightarrow M\left[\sqrt{(1+x)^2+y^2}\right] = \sqrt{x^2+y^2}$$

$$\Rightarrow M^2\left[(1+x)^2+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2\left[1+x^2+2x+y^2\right] = x^2+y^2$$

$$\Rightarrow M^2+M^2x^2+2M^2x+M^2y^2 = x^2+y^2$$

$$\Rightarrow (1-M^2)x^2 + (1-M^2)y^2 - 2xM^2 = M^2$$

$$\Rightarrow x^2 + y^2 - \frac{2M^2}{1-M^2}x = \frac{M^2}{1-M^2}$$

$$\Rightarrow \left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \frac{M^2}{1-M^2} + \left(\frac{M^2}{1-M^2}\right)^2$$

$$= \frac{M^2}{1-M^2}\left(1 + \frac{M^2}{1-M^2}\right)$$

$$= \frac{M^2}{1-M^2}\left(\frac{1-\cancel{M^2}+\cancel{M^2}}{1-M^2}\right) = \frac{M^2}{(1-M^2)^2}$$

$$\Rightarrow \boxed{\left(x - \frac{M^2}{1-M^2}\right)^2 + (y-0)^2 = \left(\frac{M}{1-M^2}\right)^2}$$

It represents a circle with centre at $\left(\frac{M^2}{1-M^2}, 0\right)$

& Radius $= \frac{M}{1-M^2}$

When M takes different values, the family of circles so formed are called constant M loci or constant M circles.

N - Circles

$$\therefore M(j\omega) = \frac{G(j\omega)}{1+G(j\omega)} = \frac{x+jy}{1+x+jy}$$

$$\Rightarrow \angle M(j\omega) = \alpha = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$= \tan^{-1}\left[\frac{\frac{y}{x} - \frac{y}{1+x}}{1+\frac{y}{x}\times\frac{y}{1+x}}\right]$$

$$\Rightarrow \tan\alpha = N = \frac{\frac{y(1+x) - yx}{x(1+x)}}{\frac{x(1+x)+y^2}{x(1+x)}}$$

$$\Rightarrow N = \frac{y}{x+x^2+y^2}$$

$$\Rightarrow N(x+x^2+y^2) = y$$

$$\Rightarrow x^2+x+y^2 = \frac{y}{N}$$

$$\Rightarrow x^2+x+y^2-\frac{y}{N} = 0$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2N}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2N}\right)^2$$

$$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2N}\right)^2 = \left(\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2N}\right)^2}\right)^2$$

It represents a circle with centre $\left(-\frac{1}{2}, \frac{1}{2N}\right)$

& Radius $= \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2N}\right)^2}$

For different values of N the circles so formed are called constant N loci or constant N circles.

(b) All pass & Minimum Phase System

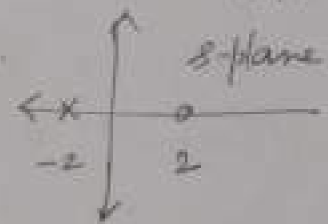
All pass system :-

(i) An all pass system has magnitude of its transfer function equal to constant for all frequency

$$\text{ie, } |H(j\omega)| = \text{constant} = k \quad \forall \omega$$

(ii) Poles must lie on the left and zeros on the mirror image of the poles on the right

ex:- $H(s) = \dfrac{s-2}{s+2}$



Put $s = j\omega$

$$H(j\omega) = \frac{j\omega - 2}{j\omega + 2}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 2^2}}{\sqrt{\omega^2 + 2^2}} = 1 \quad \forall \omega.$$

Minimum Phase System

1. A minimum phase transfer function doesnot have poles or zeros in right half s-plane, or on the $j\omega$-axis, excluding the Origin.
2. The value of minimum phase transfer function cannot become zero or infinity at any finite non-zero frequency

(c) PID Controller (Proportional-Integral-Derivative Controller)

(i) PD controller could add damping to the system, but the steady state response is not affected.
PI controller could improve the relative stability and improve the steady-state error at the same time, but the rise time is increased.

(ii) This leads to the motivation of using PID controller so that the best features of each of the PI & PD controllers are utilized.

→ PID controller as PI portion connected in cascade with a PD portion:

$$G_c(s) = K_P + K_D s + \frac{K_I}{s}$$

$$= (K_{P1} + K_{D1} s)\left(K_{P2} + \frac{K_{I2}}{s}\right)$$

$$= (K_{P1} K_{P2} + K_{D1} K_{I2}) + K_{D1} K_{P2} s + \frac{K_{P1} K_{I2}}{s}$$

$$\therefore \quad K_P = K_{P1} K_{P2} + K_{D1} K_{I2}$$

$$K_D = K_{D1} K_{P2}$$

$$K_I = K_{P1} K_{I2}$$